

CHAPTER 2 SOLUTIONS**Problem 2.1**

From Ohm's law, the current I_1 through R_1 is given by

$$I_1 = \frac{V}{R_1} = \frac{6V}{3k\Omega} = \frac{6V}{3000\Omega} = 0.002A = 2mA$$

Notice that $1V/1k\Omega = 1mA$.

From Ohm's law, the current I_2 through R_2 is given by

$$I_2 = \frac{V}{R_2} = \frac{6V}{6k\Omega} = \frac{6V}{6000\Omega} = 0.001A = 1mA$$

Problem 2.2

From Ohm's law, the current I_1 through R_1 is given by

$$I_1 = \frac{V_1}{R_1} = \frac{2.4V}{800\Omega} = 0.003A = 3mA$$

From Ohm's law, the current I_2 through R_2 is given by

$$I_2 = \frac{V_2}{R_2} = \frac{3.6V}{2k\Omega} = 1.8mA$$

From Ohm's law, the current I_3 through R_3 is given by

$$I_3 = \frac{V_2}{R_3} = \frac{3.6V}{3k\Omega} = 1.2mA$$

Problem 2.3

From Ohm's law, the current I_1 through R_1 is given by

$$I_1 = \frac{V_1}{R_1} = \frac{2.4V}{4k\Omega} = 0.6mA = 600\mu A$$

From Ohm's law, the current I_2 through R_2 is given by

$$I_2 = \frac{V_1}{R_2} = \frac{2.4V}{6k\Omega} = 0.4mA = 400\mu A$$

From Ohm's law, the current I_3 through R_3 is given by

$$I_3 = \frac{V_2}{R_2} = \frac{1.2V}{1.8k\Omega} = \frac{2}{3}mA = 0.6667mA = 666.5557\mu A$$

From Ohm's law, the current I_4 through R_4 is given by

$$I_4 = \frac{V_2}{R_4} = \frac{1.2V}{6k\Omega} = 0.2mA = 200\mu A$$

From Ohm's law, the current I_5 through R_5 is given by

$$I_5 = \frac{V_2}{R_5} = \frac{1.2V}{9k\Omega} = \frac{2}{15}mA = 0.1333mA = 133.3333\mu A$$

Problem 2.4

From Ohm's law, the voltage across R_2 is given by

$$V_o = R_2 I_2 = 6k\Omega \times 1.2mA = 6000 \times 0.0012 = 7.2V$$

Notice that $1k\Omega \times 1mA = 1V$.

From Ohm's law, the current I_1 through R_1 is given by

$$I_1 = \frac{V_1}{R_1} = \frac{2.8V}{1.4k\Omega} = 2mA$$

From Ohm's law, the voltage across R_2 is given by

$$V_o = R_2 I_2 = 6k\Omega \times 1.2mA = 6000 \times 0.0012 = 7.2V$$

From Ohm's law, the current I_3 through R_3 is given by

$$I_3 = \frac{V_o}{R_3} = \frac{7.2V}{9k\Omega} = 0.8mA = 800\mu A$$

Problem 2.5

From Ohm's law, the voltage across R_4 is given by

$$V_o = R_4 I_4 = 18k\Omega \times 0.2mA = 18000 \times 0.0002 = 3.6V$$

From Ohm's law, the current I_3 through R_3 is given by

$$I_3 = \frac{V_o}{R_3} = \frac{3.6V}{6k\Omega} = 0.6mA = 600\mu A$$

Problem 2.6

From Ohm's law, the voltage across R_4 is given by

$$V_o = R_4 I_4 = 8k\Omega \times 0.4mA = 8000 \times 0.0004 = 3.2V$$

From Ohm's law, the current I_2 through R_2 is given by

$$I_2 = \frac{V_o}{R_2} = \frac{3.2V}{3k\Omega} = \frac{16}{15}mA = 1.06667mA$$

From Ohm's law, the current I_3 through R_3 is given by

$$I_3 = \frac{V_o}{R_3} = \frac{3.2V}{6k\Omega} = \frac{16}{30}mA = 0.53333mA = 533.3333\mu A$$

Problem 2.7

From Ohm's law, the voltage across R_3 is given by

$$V_o = R_3 I_3 = 42k\Omega \times (1/12)mA = 42/12V = 3.5V$$

From Ohm's law, the resistance value R_2 is given by

$$R_2 = \frac{V_o}{I_2} = \frac{3.5V}{\frac{7}{60}mA} = 30k\Omega$$

$$1V/1mA = 1k\Omega$$

Problem 2.8

The power on R_1 is

$$P_{R_1} = I^2 R_1 = (2 \times 10^{-3})^2 \times 2000 = 4 \times 10^{-6} \times 2 \times 10^3 = 8 \times 10^{-3}W = 8mW \text{ (absorbed)}$$

The power on R_2 is

$$P_{R_2} = I^2 R_1 = (2 \times 10^{-3})^2 \times 3000 = 4 \times 10^{-6} \times 3 \times 10^3 = 12 \times 10^{-3}W = 12mW \text{ (absorbed)}$$

The power on V_s is

$$P_{V_s} = -IV_s = -2 \times 10^{-3} \times 10 = -20 \times 10^{-3} W = -20 mW \text{ (released)}$$

Total power absorbed = 20 mW = total power released

Problem 2.9

The power on R_1 is

$$P_{R_1} = \frac{V_o^2}{R_1} = \frac{4.8^2}{8000} = 2.88 \times 10^{-3} W = 2.88 mW \text{ (absorbed)}$$

The power on R_2 is

$$P_{R_2} = \frac{V_o^2}{R_2} = \frac{4.8^2}{12000} = 1.92 \times 10^{-3} W = 1.92 mW \text{ (absorbed)}$$

The power on V_s is

$$P_{I_s} = -I_s V_o = -1 \times 10^{-3} \times 4.8 = -4.8 \times 10^{-3} W = -4.8 mW \text{ (released)}$$

Problem 2.10

From Ohm's law, current I_1 is given by

$$I_1 = \frac{20V - 15V}{R_1} = \frac{5V}{0.5 k\Omega} = 10 mA$$

From Ohm's law, current I_2 is given by

$$I_2 = \frac{20V - 10V}{R_2} = \frac{10V}{2 k\Omega} = 5 mA$$

From Ohm's law, current I_3 is given by

$$I_3 = \frac{10V - 0V}{R_3} = \frac{10V}{1 k\Omega} = 10 mA$$

From Ohm's law, current I_4 is given by

$$I_4 = \frac{10V - 15V}{R_4} = \frac{-5V}{1k\Omega} = -5mA$$

Problem 2.11

From Ohm's law, current i is given by

$$i = \frac{10V - 8V}{R_3} = \frac{2V}{2k\Omega} = 1mA$$

From Ohm's law, current I_1 is given by

$$I_1 = \frac{12V - 10V}{R_1} = \frac{2V}{1k\Omega} = 2mA$$

From Ohm's law, current I_2 is given by

$$I_2 = \frac{10V - 5V}{R_2} = \frac{5V}{5k\Omega} = 1mA$$

From Ohm's law, current I_3 is given by

$$I_3 = \frac{12V - 8V}{R_4} = \frac{4V}{2k\Omega} = 2mA$$

From Ohm's law, current I_4 is given by

$$I_4 = \frac{8V - 5V}{R_5} = \frac{3V}{3k\Omega} = 1mA$$

From Ohm's law, current I_5 is given by

$$I_5 = \frac{8V}{R_6} = \frac{8V}{4k\Omega} = 2mA$$

Problem 2.12

Application of Ohm's law results in

$$I_1 = \frac{34V - 24V}{R_1} = \frac{10V}{2k\Omega} = 5mA$$

$$I_2 = \frac{24V - 10V}{R_2} = \frac{14V}{2k\Omega} = 7mA$$

$$I_3 = \frac{24V - 28V}{R_3} = \frac{-4V}{2k\Omega} = -2mA$$

$$I_4 = \frac{34V - 28V}{R_4} = \frac{6V}{0.6k\Omega} = 10mA$$

$$I_5 = \frac{28V - 10V}{R_5} = \frac{18V}{6k\Omega} = 3mA$$

$$I_6 = \frac{28V}{R_6} = \frac{28V}{5.6k\Omega} = 5mA$$

$$I_7 = \frac{10V}{R_7} = \frac{10V}{1k\Omega} = 10mA$$

Problem 2.13

The total voltage from the four voltage sources is

$$V = V_{s1} + V_{s2} + V_{s3} + V_{s4} = 9V + 2V - 3V + 2V = 10V$$

The total resistance from the five resistors is

$$R = R_1 + R_2 + R_3 + R_4 + R_5 = 3k\Omega + 5k\Omega + 4k\Omega + 2k\Omega + 4k\Omega = 18k\Omega$$

The current through the mesh is

$$I = \frac{V}{R} = \frac{10V}{18000\Omega} = \frac{5}{9}mA = 0.5556mA$$

From Ohm's law, the voltages across the five resistors are given respectively

$$V_1 = R_1 I = 3 \times 5/9 V = 15/9 V = 5/3 V = 1.6667 V$$

$$V_2 = R_2 I = 5 \times 5/9 V = 25/9 V = 2.7778 V$$

$$V_3 = R_3 I = 4 \times 5/9 V = 20/9 V = 2.2222 V$$

$$V_4 = R_4 I = 2 \times 5/9 V = 10/9 V = 1.1111 V$$

$$V_5 = R_5 I = 4 \times 5/9 \text{ V} = 20/9 \text{ V} = 2.2222 \text{ V}$$

Problem 2.14

Radius is $r = d/2 = 0.2025 \text{ mm} = 0.2025 \times 10^{-3} \text{ m}$

$$A = \pi r^2 = 1.28825 \times 10^{-7} \text{ m}^2$$

(a)

$$R = \frac{\ell}{\sigma A} = \frac{20}{5.69 \times 10^7 \times \pi \times (0.2025 \times 10^{-3})^2} = 2.7285 \Omega$$

(b)

$$R = \frac{\ell}{\sigma A} = \frac{200}{5.69 \times 10^7 \times \pi \times (0.2025 \times 10^{-3})^2} = 27.2846 \Omega$$

(c)

$$R = \frac{\ell}{\sigma A} = \frac{2000}{5.69 \times 10^7 \times \pi \times (0.2025 \times 10^{-3})^2} = 272.8461 \Omega$$

(d)

$$R = \frac{\ell}{\sigma A} = \frac{20000}{5.69 \times 10^7 \times \pi \times (0.2025 \times 10^{-3})^2} = 2728.4613 \Omega$$

Problem 2.15

From Ohm's law, the voltage across R_2 is given by

$$V_2 = I_2 R_2 = 3 \text{ mA} \times 2 \text{ k}\Omega = 6 \text{ V}$$

From Ohm's law, the current through R_3 is given by

$$I_3 = \frac{V_2}{R_3} = \frac{6 \text{ V}}{3 \text{ k}\Omega} = 2 \text{ mA}$$

According to KCL, current I_1 is the sum of I_2 and I_3 . Thus, we have

$$I_1 = I_2 + I_3 = 3 \text{ mA} + 2 \text{ mA} = 5 \text{ mA}$$

The voltage across R_1 is given by

$$V_1 = R_1 I_1 = 1 \text{ k}\Omega \times 5 \text{ mA} = 5 \text{ V}$$

Problem 2.16

From Ohm's law, the currents I_2 , I_3 , and I_4 are given respectively by

$$I_2 = \frac{V_2}{R_2} = \frac{6V}{2k\Omega} = 3mA$$

$$I_3 = \frac{V_2}{R_3} = \frac{6V}{3k\Omega} = 2mA$$

$$I_4 = \frac{V_2}{R_4} = \frac{6V}{6k\Omega} = 1mA$$

From KCL, current I_1 is the sum of I_2 , I_3 , and I_4 . Thus, we have

$$I_1 = I_2 + I_3 + I_4 = 3mA + 2mA + 1mA = 6mA$$

The voltage across R_1 is given by

$$V_1 = R_1 I_1 = 1k\Omega \times 6mA = 6V$$

Problem 2.17

From Ohm's law, we have

$$V_2 = R_4 I_4 = 1mA \times 6k\Omega = 6V$$

From Ohm's law, the current through R_3 is given by

$$I_3 = \frac{V_2}{R_3} = \frac{6V}{3k\Omega} = 2mA$$

From KCL, I_2 is the sum of I_3 and I_4 . Thus,

$$I_2 = I_3 + I_4 = 3mA$$

From KCL, I_1 is given by

$$I_1 = I_s - I_2 = 2mA$$

From Ohm's law, the voltage across R_1 is

$$V_1 = R_1 I_1 = 4.5k\Omega \times 2mA = 9V$$

Problem 2.18

From Ohm's law, we have

$$I_3 = \frac{V_o}{R_3} = \frac{8V}{2k\Omega} = 4mA$$

$$I_4 = \frac{V_o}{R_4} = \frac{8V}{4k\Omega} = 2mA$$

$$I_1 = \frac{V_s - V_o}{R_1} = \frac{12V - 8V}{1k\Omega} = \frac{4V}{1k\Omega} = 4mA$$

$$I_2 = \frac{V_s - V_o}{R_2} = \frac{12V - 8V}{2k\Omega} = \frac{4V}{2k\Omega} = 2mA$$

As a check, $I_1 + I_2 = I_3 + I_4 = 6mA$

Problem 2.19

From Ohm's law, we have

$$I_3 = \frac{V_4}{R_4} = \frac{5V}{2.5k\Omega} = 2mA$$

$$V_3 = R_3 I_3 = 2k\Omega \times 2mA = 4V$$

$$V_2 = V_3 + V_4 = 4V + 5V = 9V$$

$$I_2 = \frac{V_2}{R_2} = 3 \frac{9V}{4k\Omega} = 3mA$$

From KCL, we have

$$I_1 = I_2 + I_3 = 5mA$$

From Ohm's law, we get

$$V_1 = R_1 I_1 = 1k\Omega \times 5mA = 5V$$

Problem 2.20

Application of KCL at node a yields

$$I_s = I_1 + I_2 + I_3$$

Solving for I_2 , we obtain

$$I_2 = I_s - I_1 - I_3 = 10 \text{ mA} - 5 \text{ mA} - 2 \text{ mA} = 3 \text{ mA}$$

Application of KCL at node b yields

$$I_1 + I_2 = I_4 + I_5$$

Solving for I_5 , we obtain

$$I_5 = I_1 + I_2 - I_4 = 5 \text{ mA} + 3 \text{ mA} - 2 \text{ mA} = 6 \text{ mA}$$

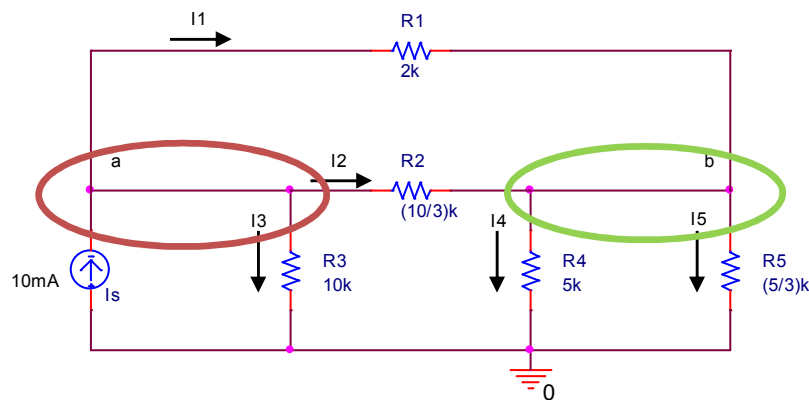


Figure S2.20

Problem 2.21

Application of KCL at node b yields

$$I_s = I_2 + I_3$$

Solving for I_2 , we obtain

$$I_2 = I_s - I_3 = 15 \text{ mA} - 10 \text{ mA} = 5 \text{ mA}$$

Application of KCL at node a yields

$$I_4 = I_2 - I_1 = 5 \text{ mA} - 2 \text{ mA} = 3 \text{ mA}$$

Application of KCL at node c yields

$$I_5 = I_1 + I_3 = 2 \text{ mA} + 10 \text{ mA} = 12 \text{ mA}$$

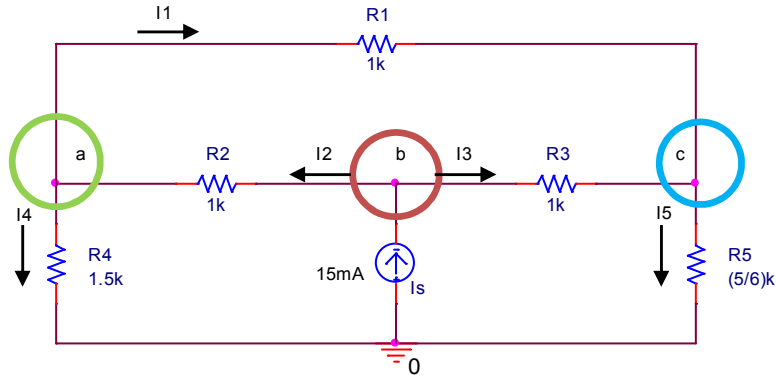


Figure S2.21

Problem 2.22

Application of KCL at node *b* yields

$$I_1 = I_s - I_4 = 20 \text{ mA} - 10 \text{ mA} = 10 \text{ mA}$$

Application of KCL at node *a* yields

$$I_2 = I_1 - I_3 = 10 \text{ mA} - 5 \text{ mA} = 5 \text{ mA}$$

Application of KCL at node *c* yields

$$I_6 = I_3 + I_4 - I_5 = 5 \text{ mA} + 10 \text{ mA} - 5 \text{ mA} = 10 \text{ mA}$$

Application of KCL at node *d* yields

$$I_7 = I_2 + I_5 = 5 \text{ mA} + 5 \text{ mA} = 10 \text{ mA}$$

Problem 2.23

Application of KCL at node *d* yields

$$I_2 = 13 - 10 = 3 \text{ A}$$

Application of KCL at node *a* yields

$$I_1 = I_2 - 2 = 3 - 2 = 1 \text{ A}$$

Application of KCL at node *b* yields

$$I_3 = -I_1 - 5 = -1 - 5 = -6 \text{ A}$$

Application of KCL at node c yields

$$I_5 = -2 - 10 = -12 \text{ A}$$

Application of KCL at node e yields

$$I_4 = -I_3 - 13 = -(-6) - 13 = -7 \text{ A}$$

Problem 2.24

Summing the voltage drops around mesh 1 in the circuit shown in Figure S2.24 in the clockwise direction, we obtain

$$-V_1 + V_{R1} + V_{R3} = 0$$

Since $V_1 = 30\text{V}$ and $V_{R1} = 10\text{V}$, this equation becomes

$$-30 + 10 + V_{R3} = 0$$

Thus,

$$V_{R3} = 30 - 10 = 20\text{V}.$$

Summing the voltage drops around mesh 2 in the circuit shown in Figure S2.11 in the clockwise direction, we obtain

$$-V_{R3} + V_{R2} + V_{R4} = 0$$

Since $V_{R3} = 20\text{V}$ and $V_{R4} = 15\text{V}$, this equation becomes

$$-20 + V_{R2} + 15 = 0$$

Thus,

$$V_{R2} = 20 - 15 = 5\text{V}.$$

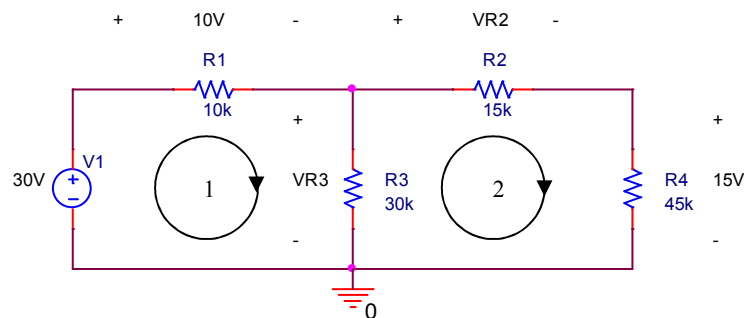


Figure S2.24

Problem 2.25

Consider the loop consisting of V_1 , R_1 and R_5 , shown in the circuit shown in Figure S2.25. Summing the voltage drops around this loop in the clockwise direction, we obtain

$$-V_1 + V_{R1} + V_{R5} = 0$$

Since $V_1 = 20\text{V}$ and $V_{R1} = 10\text{V}$, this equation becomes

$$-20 + 10 + V_{R5} = 0$$

Thus,

$$V_{R5} = 20 - 10 = 10\text{V}.$$

In the mesh consisting of R_4 , R_3 and R_5 , shown in the circuit shown in Figure S2.25, summing the voltage drops around this mesh in the clockwise direction, we obtain

$$-V_{R4} + V_{R3} + V_{R5} = 0$$

Since $V_{R3} = 5\text{V}$ and $V_{R5} = 10\text{V}$, this equation becomes

$$-V_{R4} + 5 + 10 = 0$$

Thus,

$$V_{R4} = 5 + 10 = 15\text{V}.$$

In the mesh consisting of V_1 , R_2 and R_4 , shown in the circuit shown in Figure S2.25, summing the voltage drops around this mesh in the clockwise direction, we obtain

$$-V_1 + V_{R2} + V_{R4} = 0$$

Since $V_1 = 20\text{V}$ and $V_{R4} = 15\text{V}$, this equation becomes

$$-20 + V_{R2} + 15 = 0$$

Thus,

$$V_{R2} = 20 - 15 = 5\text{V}.$$

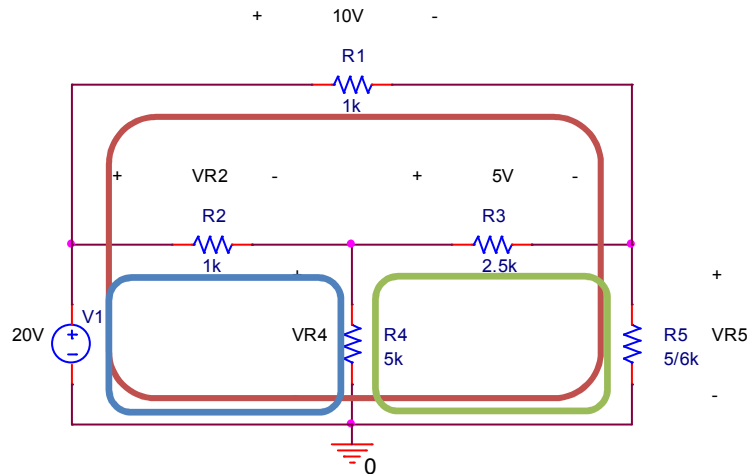


Figure S2.25

Problem 2.26

In the mesh consisting of R_1 , R_3 and R_4 , upper left in the circuit shown in Figure S2.26, summing the voltage drops around this mesh in the clockwise direction, we obtain

$$V_{R1} + V_{R3} - V_{R4} = 0$$

Since $V_{R1} = 5\text{V}$ and $V_{R3} = 5\text{V}$, this equation becomes

$$5 + 5 - V_{R4} = 0$$

Thus,

$$V_{R4} = 5 + 5 = 10\text{V}.$$

In the mesh consisting of V_1 , R_4 and R_6 , lower left in the circuit shown in Figure S2.26, summing the voltage drops around this mesh in the clockwise direction, we obtain

$$-V_1 + V_{R4} + V_{R6} = 0$$

Since $V_1 = 20\text{V}$ and $V_{R4} = 10\text{V}$, this equation becomes

$$-20 + 10 + V_{R6} = 0$$

Thus,

$$V_{R6} = 20 - 10 = 10\text{V}.$$

In the mesh consisting of R_3 , R_2 and R_5 , upper right in the circuit shown in Figure S2.26, summing the voltage drops around this mesh in the clockwise direction, we obtain

$$-V_{R3} + V_{R2} - V_{R5} = 0$$

Since $V_{R3} = 5V$ and $V_{R5} = 5V$, this equation becomes

$$-5 + V_{R2} - 5 = 0$$

Thus,

$$V_{R2} = 5 + 5 = 10V.$$

In the mesh consisting of R_6 , R_5 and R_7 , lower right in the circuit shown in Figure S2.26, summing the voltage drops around this mesh in the clockwise direction, we obtain

$$-V_{R6} + V_{R5} + V_{R7} = 0$$

Since $V_{R6} = 10V$ and $V_{R5} = 5V$, this equation becomes

$$-10 + 5 + V_{R7} = 0$$

Thus,

$$V_{R7} = 10 - 5 = 5V.$$

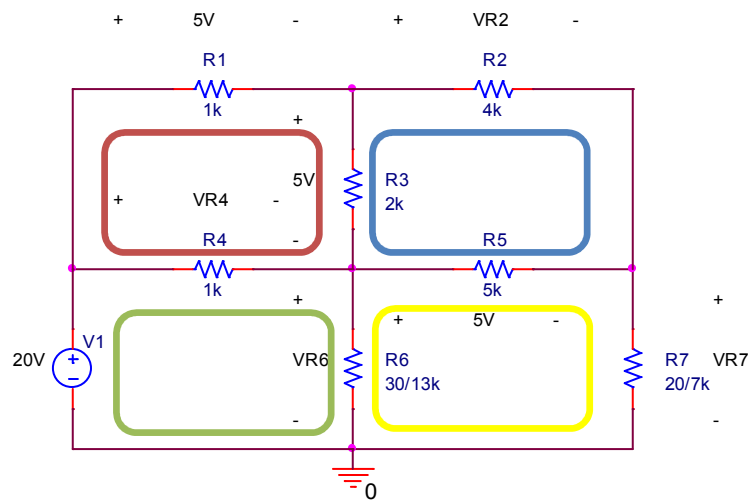


Figure S2.26

Problem 2.27

From Ohm's law, the current I_5 is given by

$$I_5 = \frac{V_5}{R_5} = \frac{6V}{1k\Omega} = 6mA$$

From Ohm's law, the current I_1 is given by

$$I_1 = \frac{V_s - V_5}{R_1} = \frac{16V - 6V}{5k\Omega} = \frac{10V}{5k\Omega} = 2mA$$

From KCL, we have

$$I_3 = I_5 - I_1 = 6mA - 2mA = 4mA$$

The voltage across R_3 is

$$V_3 = R_3 I_3 = 1k\Omega \times 4mA = 4V$$

From KVL, the voltage across R_4 is given by

$$V_4 = V_3 + V_5 = 4V + 6V = 10V$$

The current through R_4 is given by

$$I_4 = \frac{V_4}{R_4} = \frac{10V}{5k\Omega} = 2mA$$

From KCL, current I_2 is given by

$$I_2 = I_3 + I_4 = 4mA + 2mA = 6mA$$

Problem 2.28

The voltage across R_3 is given by

$$V_2 = R_3 I_3 = 4k\Omega \times 2mA = 8V$$

From Ohm's law, current I_4 is given by

$$I_4 = \frac{V_2}{R_4} = \frac{8V}{2k\Omega} = 4mA$$

From KCL, the current through R_2 is given by

$$I_2 = I_3 + I_4 = 2mA + 4mA = 6mA$$

From KCL, the current through R_1 is given by

$$I_1 = I_s - I_2 = 8mA - 6mA = 2mA$$

The voltage across R_1 is given by

$$V_1 = R_1 I_1 = 7 \text{ k}\Omega \times 2 \text{ mA} = 14 \text{ V}$$

Problem 2.29

The voltage across R_1 is given by

$$V_1 = R_1 I_1 = 5 \text{ k}\Omega \times 1 \text{ mA} = 5 \text{ V}$$

From KCL, the current through R_2 is given by

$$I_2 = I_s - I_1 = 5 \text{ mA} - 1 \text{ mA} = 4 \text{ mA}$$

From KVL, V_2 is given by

$$V_2 = V_1 - R_2 I_2 = 5 \text{ V} - 0.5 \text{ k}\Omega \times 4 \text{ mA} = 5 \text{ V} - 2 \text{ V} = 3 \text{ V}$$

From Ohm's law, current I_3 is given by

$$I_3 = \frac{V_2}{R_3} = \frac{3 \text{ V}}{1 \text{ k}\Omega} = 3 \text{ mA}$$

From Ohm's law, current I_4 is given by

$$I_4 = \frac{V_2}{R_4} = \frac{3 \text{ V}}{3 \text{ k}\Omega} = 1 \text{ mA}$$

Problem 2.30

Application of KVL around the outer loop yields

$$-2 - V_1 - 3 = 0$$

Solving for V_1 , we obtain

$$V_1 = -5 \text{ V}$$

Application of KVL around the top mesh yields

$$-V_1 - 4 + V_2 = 0$$

Solving for V_2 , we obtain

$$V_2 = V_1 + 4 = -1 \text{ V}$$

Application of KVL around the center left mesh yields

$$-V_2 + 5 - V_3 = 0$$

Solving for V_3 , we obtain

$$V_3 = -V_2 + 5 = 6 \text{ V}$$

Application of KVL around the center right mesh yields

$$-5 + 4 + V_4 = 0$$

Solving for V_4 , we obtain

$$V_4 = 5 - 4 = 1 \text{ V}$$

Application of KVL around the bottom left mesh yields

$$-2 + V_3 - V_5 = 0$$

Solving for V_5 , we obtain

$$V_5 = -2 + 6 = 4 \text{ V}$$

Problem 2.31

Application of KVL around the outer loop yields

$$-3 - V_1 = 0$$

Solving for V_1 , we obtain

$$V_1 = -3 \text{ V}$$

Application of KVL around the lower left mesh yields

$$-3 + V_2 - 1 = 0$$

Solving for V_2 , we obtain

$$V_2 = 3 + 1 = 4 \text{ V}$$

Application of KVL around the lower right mesh yields

$$1 - V_5 = 0$$

Solving for V_5 , we obtain

$$V_5 = 1 \text{ V}$$

Application of KCL at node a yields

$$I_1 = 2 + 2 = 4 \text{ A}$$

Application of KCL at node b yields

$$I_4 = 2 + 3 = 5 \text{ A}$$

Problem 2.32

Resistor R_1 is in series to the parallel combination of R_2 and R_3 . Thus, the equivalent resistance R_{eq} is given by

$$\begin{aligned} R_{eq} &= R_1 + (R_2 \parallel R_3) = R_1 + \frac{R_2 R_3}{R_2 + R_3} = 2000 + \frac{4000 \times 12000}{4000 + 12000} \\ &= 2000 + \frac{48,000,000}{16,000} = 2000 + 3000 = 5000 \Omega = 5 \text{ k}\Omega \end{aligned}$$

Instead of ohms (Ω), we can use kilo ohms ($\text{k}\Omega$) to simplify the algebra:

$$R_{eq} = R_1 + (R_2 \parallel R_3) = R_1 + \frac{R_2 R_3}{R_2 + R_3} = 2\text{k} + \frac{4\text{k} \times 12\text{k}}{4\text{k} + 12\text{k}} = 2\text{k} + \frac{48\text{k}^2}{16\text{k}} = 2\text{k} + 3\text{k} = 5\text{k}\Omega$$

If all the resistance values are in $\text{k}\Omega$, k can be removed during calculations, and represent the answer in $\text{k}\Omega$ as shown below.

$$R_{eq} = R_1 + (R_2 \parallel R_3) = R_1 + \frac{R_2 R_3}{R_2 + R_3} = 2 + \frac{4 \times 12}{4 + 12} = 2 + \frac{48}{16} = 2 + 3 = 5 \text{ k}\Omega$$

Problem 2.33

Resistors R_1 and R_2 are in parallel, and resistors R_3 and R_4 are in parallel. The equivalent resistance is the sum of $R_1 \parallel R_2$ and $R_3 \parallel R_4$.

$$R_{eq} = (R_1 \parallel R_2) + (R_3 \parallel R_4) = \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4} = \frac{10 \times 40}{10 + 40} + \frac{8 \times 56}{8 + 56} = \frac{400}{50} + \frac{448}{64} = 8 + 7 = 15 \text{ k}\Omega$$

Problem 2.34

The equivalent resistance is the sum of R_1 and the parallel combination of R_2 , R_3 , and R_4 .

$$R_{eq} = R_1 + (R_2 \parallel R_3 \parallel R_4) = R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} = 5 + \frac{1}{\frac{1}{30} + \frac{1}{60} + \frac{1}{5}} = 5 + \frac{1}{\frac{2}{60} + \frac{1}{60} + \frac{12}{60}}$$

$$= 5 + \frac{60}{15} = 5 + 4 = 9 \text{ k}\Omega$$

Problem 2.35

The equivalent resistance of the parallel combination of R_4 and a short circuit (0Ω) is given by

$$R_4 \parallel 0 = \frac{20 \times 0}{20 + 0} = \frac{0}{20} = 0 \Omega$$

The equivalent resistance is the sum of R_1 and the parallel combination of R_2 and R_3 .

$$R_{eq} = R_1 + (R_2 \parallel R_3) = R_1 + \frac{R_2 R_3}{R_2 + R_3} = 12 + \frac{99 \times 22}{99 + 22} = 12 + \frac{2178}{121} = 12 + 18 = 30 \text{ k}\Omega$$

Problem 2.36

The equivalent resistance R_a of the series connection of three resistors R_4 , R_5 , and R_6 is

$$R_a = R_4 + R_5 + R_6 = 25 + 20 + 33 = 78 \text{ k}\Omega$$

The equivalent resistance R_b of the parallel connection of R_3 and R_a is

$$R_b = R_3 \parallel R_a = \frac{R_3 R_a}{R_3 + R_a} = \frac{39 \times 78}{39 + 78} = \frac{3042}{117} = 26 \text{ k}\Omega$$

The equivalent resistance R_{eq} of the circuit shown in Figure P2.5 is the sum of R_1 , R_b , and R_2 :

$$R_{eq} = R_1 + R_b + R_2 = 10 + 26 + 14 = 50 \text{ k}\Omega$$

Problem 2.37

The resistors R_1 and R_2 are connected in parallel. Let R_a be $R_1 \parallel R_2$. Then, we have

$$R_a = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{50 \times 75}{50 + 75} = \frac{50 \times 75}{125} = \frac{50 \times 3}{5} = 10 \times 3 = 30 \text{ k}\Omega$$

The resistors R_3 and R_4 are connected in parallel. Let R_b be $R_3 \parallel R_4$. Then, we have

$$R_b = R_3 \parallel R_4 = \frac{R_3 R_4}{R_3 + R_4} = \frac{55 \times 66}{55 + 66} = \frac{5 \times 66}{5 + 6} = \frac{5 \times 66}{11} = 5 \times 6 = 30 \text{ k}\Omega$$

The equivalent resistance R_{eq} of the circuit shown in Figure P2.6 is given by the sum of R_a and R_b :

$$R_{eq} = R_a + R_b = 30 \text{ k}\Omega + 30 \text{ k}\Omega = 60 \text{ k}\Omega$$

MATLAB

```
clear all;
R1=50000;R2=75000;R3=55000;R4=66000;
Req=P([R1,R2])+P([R3,R4])
```

Answer:

```
Req =
    60000
```

Problem 2.38

The equivalent resistance R_{eq} can be found by combining resistances from the right side of the circuit and moving toward the left. Since R_7 , R_8 , and R_9 are connected in series, we have

$$R_a = R_7 + R_8 + R_9 = 15 + 19 + 20 = 54 \text{ k}\Omega$$

Let R_b be the equivalent resistance of the parallel connection of R_6 and R_a . Then we have

$$R_b = R_6 \parallel R_a = \frac{R_6 \times R_a}{R_6 + R_a} = \frac{27 \times 54}{27 + 54} = \frac{1 \times 54}{1 + 2} = \frac{54}{3} = 18 \text{ k}\Omega$$

Let R_c be the sum of R_4 , R_b , and R_5 . Then, we have

$$R_c = R_4 + R_b + R_5 = 6 + 18 + 4 = 28 \text{ k}\Omega.$$

Let R_d be the equivalent resistance of the parallel connection of R_3 and R_c . Then, we have

$$R_d = R_3 \parallel R_c = \frac{R_3 \times R_c}{R_3 + R_c} = \frac{21 \times 28}{21 + 28} = \frac{3 \times 28}{3 + 4} = \frac{3 \times 28}{7} = 12 \text{ k}\Omega$$

The equivalent resistance R_{eq} is the sum of R_1 , R_d , and R_2 . Thus, we have

$$R_{eq} = R_1 + R_d + R_2 = 3 + 12 + 5 = 20 \text{ k}\Omega$$

MATLAB

```
clear all;
R1=3000;R2=5000;R3=21000;R4=6000;R5=4000;R6=27000;R7=15000;R8=19000;R9=20000;
Req=R1+R2+P([R3,R4+R5+P([R6,R7+R8+R9])])
```

Answer:

```
Req =
    20000
```

Problem 2.39

Let R_a be the equivalent resistance of the parallel connection of R_5 and R_6 . Then, we have

$$R_a = R_5 \parallel R_6 = \frac{R_5 \times R_6}{R_5 + R_6} = \frac{20 \times 20}{20 + 20} = \frac{1 \times 20}{1 + 1} = \frac{20}{2} = 10 \text{ k}\Omega$$

Let R_b be the equivalent resistance of the series connection of R_4 and R_a . Then, we have

$$R_b = R_4 + R_a = 10 + 10 = 20 \text{ k}\Omega.$$

Let R_c be the equivalent resistance of the parallel connection of R_3 and R_b . Then, we have

$$R_c = R_3 \parallel R_b = \frac{R_3 \times R_b}{R_3 + R_b} = \frac{20 \times 20}{20 + 20} = \frac{1 \times 20}{1 + 1} = \frac{20}{2} = 10 \text{ k}\Omega$$

Let R_d be the equivalent resistance of the series connection of R_2 and R_c . Then, we have

$$R_d = R_2 + R_c = 10 + 10 = 20 \text{ k}\Omega.$$

The equivalent resistance R_{eq} of the circuit shown in Figure P3.8 is the parallel connection of R_1 and R_d . Thus, we get

$$R_{eq} = R_1 \parallel R_d = \frac{R_1 \times R_d}{R_1 + R_d} = \frac{20 \times 20}{20 + 20} = \frac{1 \times 20}{1 + 1} = \frac{20}{2} = 10 \text{ k}\Omega$$

MATLAB

```
clear all;
R1=20000;R2=10000;R3=20000;R4=10000;R5=20000;R6=20000;
Req=P([R1,R2+P([R3,R4+P([R5,R6])])])
```

Answer:

```
Req =
    10000
```

Problem 2.40

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} = \frac{1}{\frac{1}{2000} + \frac{1}{5000} + \frac{1}{4000} + \frac{1}{3000}} = \frac{60000}{30+12+15+20}$$

$$= \frac{60000}{77} = 779.2208\Omega$$

```
>> R1=2000;R2=5000;R3=4000;R4=3000;
>> Req=P([R1,R2,R3,R4])
Req =
    7.792207792207792e+02
```

Problem 2.41

Let $R_9 = R_2 \parallel R_3 \parallel R_4$, $R_{10} = R_6 \parallel R_7 \parallel R_8$, and $R_{11} = R_9 + R_5 + R_{10}$. Then, $R_{eq} = R_1 \parallel R_{11}$.

$$R_9 = \frac{1}{\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} = \frac{1}{\frac{1}{1000} + \frac{1}{2700} + \frac{1}{2000}} = 534.6535\Omega$$

$$R_{10} = \frac{1}{\frac{1}{R_6} + \frac{1}{R_7} + \frac{1}{R_8}} = \frac{1}{\frac{1}{2000} + \frac{1}{1500} + \frac{1}{6000}} = 750\Omega$$

$$R_{11} = R_9 + R_5 + R_{10} = 3.7837\text{ k}\Omega$$

$$R_{eq} = \frac{R_1 R_{11}}{R_1 + R_{11}} = 2.877215\text{ k}\Omega$$

```
clear all;
R1=12000;R2=1000;R3=2700;R4=2000;R5=2500;R6=2000;R7=1500;R8=6000;
R9=P([R2,R3,R4])
R10=P([R6,R7,R8])
R11=R9+R5+R10
Req=P([R1,R11])
```

Answer:

```
Req =
    2.877214991375255e+03
```

Problem 2.42

Let $R_6 = R_1 \parallel R_2$, $R_7 = R_3 \parallel R_4$. Then we have

$$R_6 = \frac{R_1 R_2}{R_1 + R_2} = 571.4286\Omega$$

$$R_7 = \frac{R_3 R_4}{R_3 + R_4} = 1.66667 \text{ k}\Omega$$

$$R_{\text{eq}} = R_6 + R_7 + R_5 = 2.7381 \text{ k}\Omega$$

```
clear all;
R1=600;R2=12000;R3=2000;R4=10000;R5=500;
R6=P([R1,R2])
R7=P([R3,R4])
Req=R6+R7+R5
```

Answer:

```
Req =
    2.738095238095238e+03
```

Problem 2.43

Let $R_9 = R_3 \parallel R_4$, $R_{10} = R_5 \parallel R_6$, $R_{11} = R_7 \parallel R_8$, $R_{12} = R_2 + R_9$, $R_{13} = R_{10} + R_{11}$. Then,
 $R_{\text{eq}} = R_1 + (R_{12} \parallel R_{13})$.

$$R_9 = \frac{R_3 \times R_4}{R_3 + R_4} = \frac{60k \times 20k}{60k + 20k} = \frac{1200k}{80} = 15k\Omega$$

$$R_{10} = \frac{R_5 \times R_6}{R_5 + R_6} = \frac{10k \times 15k}{10k + 15k} = \frac{150k}{25} = 6k\Omega$$

$$R_{11} = \frac{R_7 \times R_8}{R_7 + R_8} = \frac{20k \times 30k}{20k + 30k} = \frac{600k}{50} = 12k\Omega$$

$$R_{12} = R_2 + R_9 = 3k\Omega + 15k\Omega = 18k\Omega$$

$$R_{13} = R_{10} + R_{11} = 6k\Omega + 12k\Omega = 18k\Omega$$

$$R_{\text{eq}} = R_1 + (R_{12} \parallel R_{13}) = 6k\Omega + (18k\Omega \parallel 18k\Omega) = 6k\Omega + 9k\Omega = 15k\Omega$$

```
clear all;
R1=6000;R2=3000;R3=60000;R4=20000;R5=10000;R6=15000;R7=20000;R8=30000;
R9=P([R3,R4])
R10=P([R5,R6])
R11=P([R7,R8])
R12=R2+R9
R13=R10+R11
Req=R1+P([R12,R13])
```

```
Req =
    15000
```

Problem 2.44

Let $R_6 = R_4 \parallel R_5$, $R_7 = R_3 + R_6$, $R_8 = R_2 \parallel R_7$. Then, $R_{\text{eq}} = R_1 + R_8$.

$$R_6 = \frac{R_4 \times R_5}{R_4 + R_5} = \frac{2k \times 3k}{2k + 3k} = \frac{6k}{5} = 1.2k\Omega$$

$$R_7 = R_3 + R_6 = 1.8k\Omega + 1.2k\Omega = 3k\Omega$$

$$R_8 = \frac{R_2 \times R_7}{R_2 + R_7} = \frac{7k \times 3k}{7k + 3k} = \frac{21k}{10} = 2.1k\Omega$$

$$R_{eq} = R_1 + R_8 = 0.9k\Omega + 2.1k\Omega = 3k\Omega$$

```
clear all;
R1=900;R2=7000;R3=1800;R4=2000;R5=3000;
Req=R1+P([R2,R3+P([R4,R5])])
```

```
Answer:
Req =
    3000
```

Problem 2.45

Let $R_8 = R_6 \parallel R_7$, $R_9 = R_4 + R_5 + R_8$. Then, $R_{eq} = R_1 \parallel R_2 \parallel R_3 \parallel R_9$.

$$R_8 = \frac{R_6 \times R_7}{R_6 + R_7} = \frac{20k \times 80k}{20k + 80k} = \frac{1600k}{100} = 16k\Omega$$

$$R_9 = R_4 + R_5 + R_8 = 10k\Omega + 4k\Omega + 16k\Omega = 30k\Omega$$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_9}} = \frac{1}{\frac{1}{4000} + \frac{1}{10000} + \frac{1}{30000} + \frac{1}{30000}} = 2.4k\Omega$$

```
clear all;
R1=4000;R2=10000;R3=30000;R4=10000;R5=4000;R6=20000;R7=80000;
Req=P([R1,R2,R3,R4+R5+P([R6,R7])])
```

```
Req =
    2400
```

Problem 2.46

Let $R_8 = R_3 \parallel R_4$, $R_9 = R_6 \parallel R_7$, $R_{10} = R_8 + R_5 + R_9$. Then, $R_{eq} = R_1 \parallel R_2 \parallel R_{10}$.

$$R_8 = \frac{R_3 \times R_4}{R_3 + R_4} = \frac{10k \times 10k}{10k + 10k} = \frac{100k}{20} = 5k\Omega$$

$$R_9 = \frac{R_6 \times R_7}{R_6 + R_7} = \frac{10k \times 15k}{10k + 15k} = \frac{150k}{25} = 6k\Omega$$

$$R_{10} = R_8 + R_5 + R_9 = 5k\Omega + 4k\Omega + 6k\Omega = 15k\Omega$$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{10}}} = \frac{1}{\frac{1}{3000} + \frac{1}{10000} + \frac{1}{15000}} = \frac{30000}{\frac{30000}{3000} + \frac{30000}{10000} + \frac{30000}{15000}} = \frac{30000}{15} = 2k\Omega$$

```
clear all;
R1=3000;R2=10000;R3=10000;R4=10000;R5=4000;R6=10000;R7=15000;
Req=P([R1,R2,P([R3,R4])]+R5+P([R6,R7]))
```

```
Req =
    2000
```

Problem 2.47

The voltage from the voltage source is divided into V_1 and V_2 in proportion to the resistance values. Thus, we have

$$V_1 = \frac{R_1}{R_1 + R_2} V_s = \frac{2.5}{2.5 + 7.5} 20 V = \frac{1}{4} 20 V = 5 V$$

$$V_2 = \frac{R_2}{R_1 + R_2} V_s = \frac{7.5}{2.5 + 7.5} 20 V = \frac{3}{4} 20 V = 15 V$$

Notice that V_2 can also be obtained from $V_2 = V_s - V_1 = 20 - 5 = 15 V$.

Problem 2.48

The equivalent resistance of the parallel connection of R_2 and R_3 is given by

$$R_4 = R_2 \parallel R_3 = \frac{R_2 R_3}{R_2 + R_3} = \frac{38k \times 57k}{38k + 57k} = \frac{2166}{95} k = 22.8 k\Omega$$

The voltage V_1 across R_1 is given by

$$V_1 = \frac{R_1}{R_1 + R_4} V_s = \frac{27.2}{27.2 + 22.8} 25 V = \frac{27.2}{50} 25 V = \frac{27.2}{2} V = 13.6 V$$

The voltage V_2 across R_2 and R_3 is given by

$$V_2 = \frac{R_4}{R_1 + R_4} V_s = \frac{22.8}{27.2 + 22.8} 25 V = \frac{22.8}{50} 25 V = \frac{22.8}{2} V = 11.4 V$$

Notice that V_2 can also be obtained from $V_2 = V_s - V_1 = 25 - 13.6 = 11.4 V$.

Problem 2.49

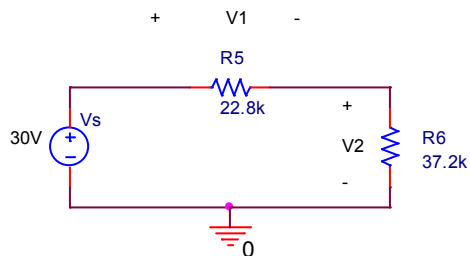
Let R_5 be the equivalent resistance of the parallel connection of R_1 and R_2 . Then, we have

$$R_5 = \frac{R_1 R_2}{R_1 + R_2} = \frac{30k \times 95k}{30k + 95k} = \frac{2850}{125} k = 22.8 \text{ k}\Omega$$

Let R_6 be the equivalent resistance of the parallel connection of R_3 and R_4 . Then, we have

$$R_6 = \frac{R_3 R_4}{R_3 + R_4} = \frac{62k \times 93k}{62k + 93k} = \frac{5766}{155} k = 37.2 \text{ k}\Omega$$

The circuit reduces to



The voltage V_1 across R_5 is given by

$$V_1 = \frac{R_5}{R_5 + R_6} V_s = \frac{22.8}{22.8 + 37.2} \times 30 \text{ V} = \frac{22.8}{60} \times 30 \text{ V} = \frac{22.8}{2} \text{ V} = 11.4 \text{ V}$$

The voltage V_2 across R_6 is given by

$$V_2 = \frac{R_6}{R_5 + R_6} V_s = \frac{37.2}{22.8 + 37.2} \times 30 \text{ V} = \frac{37.2}{60} \times 30 \text{ V} = \frac{37.2}{2} \text{ V} = 18.6 \text{ V}$$

Notice that V_2 can also be obtained from $V_2 = V_s - V_1 = 30 - 11.4 = 18.6 \text{ V}$.

Problem 2.50

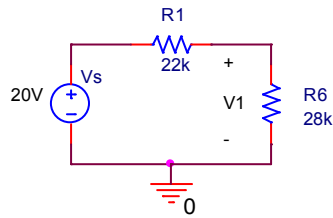
Let R_5 be the combined resistance of the series connection of R_3 and R_4 . Then, we have

$$R_5 = R_3 + R_4 = 24 \text{ k}\Omega + 60 \text{ k}\Omega = 84 \text{ k}\Omega.$$

Let R_6 be the equivalent resistance of the parallel connection of R_2 and R_5 . Then, R_6 is given by

$$R_6 = R_2 \parallel R_5 = \frac{R_2 R_5}{R_2 + R_5} = \frac{42k \times 84k}{42k + 84k} = \frac{3528}{126} k = 28 \text{ k}\Omega$$

The circuit reduces to



The voltage V_1 across R_6 is given by

$$V_1 = \frac{R_6}{R_1 + R_6} V_s = \frac{28}{22 + 28} \times 20 \text{ V} = \frac{28}{50} \times 20 \text{ V} = \frac{56}{5} \text{ V} = 11.2 \text{ V}$$

The voltage V_1 is split between R_3 and R_4 in proportion to the resistance values. Applying the voltage divider rule, we have

$$V_2 = \frac{R_4}{R_3 + R_4} V_1 = \frac{60}{24 + 60} \times 11.2 \text{ V} = \frac{60}{84} \times 11.2 \text{ V} = 8 \text{ V}$$

Problem 2.51

Let R_6 be the equivalent resistance of the parallel connection of R_4 and R_5 . Then, we have

$$R_6 = \frac{R_4 R_5}{R_4 + R_5} = \frac{22k \times 99k}{22k + 99k} = \frac{2178}{121} k = 18 \text{ k}\Omega$$

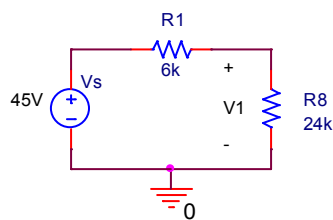
Let R_7 be the equivalent resistance of the series connection of R_3 and R_6 . Then, we have

$$R_7 = R_3 + R_6 = 70 \text{ k}\Omega + 18 \text{ k}\Omega = 88 \text{ k}\Omega.$$

Let R_8 be the equivalent resistance of the parallel connection of R_2 and R_7 . Then, we have

$$R_8 = \frac{R_2 R_7}{R_2 + R_7} = \frac{33k \times 88k}{33k + 88k} = \frac{2904}{121} k = 24 \text{ k}\Omega$$

The circuit reduces to



The voltage V_1 across R_8 is given by

$$V_1 = \frac{R_8}{R_1 + R_8} V_s = \frac{24}{6 + 24} \times 45 \text{ V} = \frac{24}{30} \times 45 \text{ V} = \frac{72}{2} \text{ V} = 36 \text{ V}$$

The voltage across R_1 is given by

$$V_{R1} = \frac{R_1}{R_1 + R_8} V_s = \frac{6}{6 + 24} \times 45 \text{ V} = \frac{6}{30} \times 45 \text{ V} = \frac{18}{2} \text{ V} = 9 \text{ V}$$

The voltage V_1 is split between R_3 and R_6 in proportion to the resistance values. Applying the voltage divider rule, we obtain

$$V_2 = \frac{R_6}{R_3 + R_6} V_1 = \frac{18}{70 + 18} \times 36 \text{ V} = \frac{18}{88} \times 36 \text{ V} = \frac{81}{11} \text{ V} = 7.3636 \text{ V}$$

The voltage across R_3 is given by

$$V_{R3} = \frac{R_3}{R_3 + R_6} V_1 = \frac{70}{70 + 18} \times 36 \text{ V} = \frac{70}{88} \times 36 \text{ V} = \frac{315}{11} \text{ V} = 28.6364 \text{ V}$$

Problem 2.52

Let R_8 be the equivalent resistance of the parallel connection of R_6 and R_7 . Then, we have

$$R_8 = \frac{R_6 R_7}{R_6 + R_7} = \frac{6 \times 12}{6 + 12} \text{ k} = \frac{72}{18} \text{ k} = 4 \text{ k}\Omega$$

Let R_9 be the equivalent resistance of the series connection of R_5 and R_8 . Then, we have

$$R_9 = R_5 + R_8 = 5 \text{ k}\Omega + 4 \text{ k}\Omega = 9 \text{ k}\Omega.$$

Let R_{10} be the equivalent resistance of the parallel connection of R_4 and R_9 . Then, we have

$$R_{10} = \frac{R_4 R_9}{R_4 + R_9} = \frac{18 \times 9}{18 + 9} \text{ k} = \frac{162}{27} \text{ k} = 6 \text{ k}\Omega$$

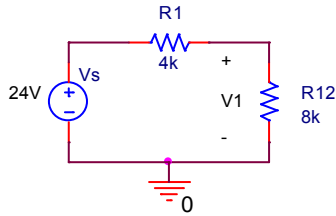
Let R_{11} be the equivalent resistance of the series connection of R_3 and R_{10} . Then, we have

$$R_{11} = R_3 + R_{10} = 4 \text{ k}\Omega + 6 \text{ k}\Omega = 10 \text{ k}\Omega.$$

Let R_{12} be the equivalent resistance of the parallel connection of R_2 and R_{11} . Then, we have

$$R_{12} = \frac{R_2 R_{11}}{R_2 + R_{11}} = \frac{40 \times 10}{40 + 10} \text{ k} = \frac{400}{50} \text{ k} = 8 \text{ k}\Omega$$

The circuit reduces to



The voltage V_1 across R_{12} is given by

$$V_1 = \frac{R_{12}}{R_1 + R_{12}} V_s = \frac{8}{4 + 8} \times 24 \text{ V} = \frac{8}{12} \times 24 \text{ V} = 16 \text{ V}$$

The voltage across R_1 is given by

$$V_{R_1} = \frac{R_1}{R_1 + R_{12}} V_s = \frac{4}{4 + 8} \times 24 \text{ V} = \frac{4}{12} \times 24 \text{ V} = 8 \text{ V}$$

The voltage V_1 is split between R_3 and R_{10} in proportion to the resistance values. Applying the voltage divider rule, we obtain

$$V_2 = \frac{R_{10}}{R_3 + R_{10}} V_1 = \frac{6}{4 + 6} \times 16 \text{ V} = \frac{6}{10} \times 16 \text{ V} = \frac{48}{5} \text{ V} = 9.6 \text{ V}$$

The voltage across R_3 is given by

$$V_{R_3} = \frac{R_3}{R_3 + R_{10}} V_1 = \frac{4}{4 + 6} \times 16 \text{ V} = \frac{4}{10} \times 16 \text{ V} = \frac{32}{5} \text{ V} = 6.4 \text{ V}$$

The voltage V_2 is split between R_5 and R_8 in proportion to the resistance values. Applying the voltage divider rule, we obtain

$$V_3 = \frac{R_8}{R_5 + R_8} V_2 = \frac{4}{5 + 4} \times 9.6 \text{ V} = \frac{4}{9} \times 9.6 \text{ V} = \frac{12.8}{3} \text{ V} = 4.2667 \text{ V}$$

The voltage across R_5 is given by

$$V_{R_5} = \frac{R_5}{R_5 + R_8} V_2 = \frac{5}{5 + 4} \times 9.6 \text{ V} = \frac{5}{9} \times 9.6 \text{ V} = \frac{16}{3} \text{ V} = 5.3333 \text{ V}$$

Problem 2.53

Let R_7 be the equivalent resistance of the parallel connection of R_4 , R_5 and R_6 . Then, we have

$$R_7 = \frac{1}{\frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6}} = \frac{1}{\frac{1}{30} + \frac{1}{36} + \frac{1}{45}} k = 12 k\Omega$$

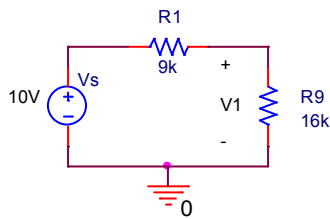
Let R_8 be the equivalent resistance of the series connection of R_3 and R_7 . Then, we have

$$R_8 = R_3 + R_7 = 8 k\Omega + 12 k\Omega = 20 k\Omega.$$

Let R_9 be the equivalent resistance of the parallel connection of R_2 and R_8 . Then, we have

$$R_9 = \frac{R_2 R_8}{R_2 + R_8} = \frac{80 \times 20}{80 + 20} k = \frac{1600}{100} k = 16 k\Omega$$

The circuit reduces to



The voltage V_1 across R_9 is given by

$$V_1 = \frac{R_9}{R_1 + R_9} V_s = \frac{16}{9 + 16} \times 10 V = \frac{16}{25} \times 10 V = \frac{32}{5} V = 6.4 V$$

The voltage across R_1 is given by

$$V_{R1} = \frac{R_1}{R_1 + R_9} V_s = \frac{9}{9 + 16} \times 10 V = \frac{9}{25} \times 10 V = \frac{18}{5} V = 3.6 V$$

The voltage V_1 is split between R_3 and R_7 in proportion to the resistance values. Applying the voltage divider rule, we obtain

$$V_2 = \frac{R_7}{R_3 + R_7} V_1 = \frac{12}{8 + 12} \times 6.4 V = \frac{12}{20} \times 6.4 V = \frac{96}{25} V = 3.84 V$$

The voltage across R_3 is given by

$$V_{R_3} = \frac{R_3}{R_3 + R_7} V_1 = \frac{8}{8+12} \times 6.4 \text{ V} = \frac{8}{20} \times 6.4 \text{ V} = \frac{64}{25} \text{ V} = 2.56 \text{ V}$$

Problem 2.54

Let R_9 be the equivalent resistance of the parallel connection of R_6 , R_7 and R_8 . Then, we have

$$R_9 = \frac{1}{\frac{1}{R_6} + \frac{1}{R_7} + \frac{1}{R_8}} = \frac{1}{\frac{1}{18} + \frac{1}{27} + \frac{1}{54}} \text{ k} = 9 \text{ k}\Omega$$

Let R_{10} be the equivalent resistance of the series connection of R_5 and R_9 . Then, we have

$$R_{10} = R_5 + R_9 = 6 \text{ k}\Omega + 9 \text{ k}\Omega = 15 \text{ k}\Omega.$$

Let R_{11} be the equivalent resistance of the parallel connection of R_4 and R_{10} . Then, we have

$$R_{11} = \frac{R_4 R_{10}}{R_4 + R_{10}} = \frac{30 \times 15}{30 + 15} \text{ k} = \frac{450}{45} \text{ k} = 10 \text{ k}\Omega$$

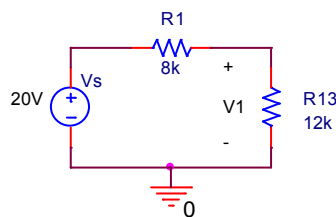
Let R_{12} be the equivalent resistance of the series connection of R_3 and R_{11} . Then, we have

$$R_{12} = R_3 + R_{11} = 10 \text{ k}\Omega + 10 \text{ k}\Omega = 20 \text{ k}\Omega.$$

Let R_{13} be the equivalent resistance of the parallel connection of R_2 and R_{12} . Then, we have

$$R_{13} = \frac{R_2 R_{12}}{R_2 + R_{12}} = \frac{30 \times 20}{30 + 20} \text{ k} = \frac{600}{50} \text{ k} = 12 \text{ k}\Omega$$

The circuit reduces to



The voltage V_1 across R_{13} is given by

$$V_1 = \frac{R_{13}}{R_1 + R_{13}} V_s = \frac{12}{8+12} \times 20 \text{ V} = \frac{12}{20} \times 20 \text{ V} = 12 \text{ V}$$

The voltage across R_1 is given by

$$V_{R1} = \frac{R_1}{R_1 + R_{13}} V_s = \frac{8}{8 + 12} \times 20 \text{ V} = \frac{8}{20} \times 20 \text{ V} = 8 \text{ V}$$

The voltage V_1 is split between R_3 and R_{11} in proportion to the resistance values. Applying the voltage divider rule, we obtain

$$V_2 = \frac{R_{11}}{R_3 + R_{11}} V_1 = \frac{10}{10 + 10} \times 12 \text{ V} = \frac{10}{20} \times 12 \text{ V} = 6 \text{ V}$$

The voltage across R_3 is given by

$$V_{R3} = \frac{R_3}{R_3 + R_{11}} V_1 = \frac{10}{10 + 10} \times 12 \text{ V} = \frac{10}{20} \times 12 \text{ V} = 6 \text{ V}$$

The voltage V_2 is split between R_5 and R_9 in proportion to the resistance values. Applying the voltage divider rule, we obtain

$$V_3 = \frac{R_9}{R_5 + R_9} V_2 = \frac{9}{6 + 9} \times 6 \text{ V} = \frac{9}{15} \times 6 \text{ V} = \frac{18}{5} \text{ V} = 3.6 \text{ V}$$

The voltage across R_5 is given by

$$V_{R5} = \frac{R_5}{R_5 + R_9} V_2 = \frac{6}{6 + 9} \times 6 \text{ V} = \frac{6}{15} \times 6 \text{ V} = \frac{12}{5} \text{ V} = 2.4 \text{ V}$$

Problem 2.55

Let R_7 be the equivalent resistance of the parallel connection of R_4 , R_5 and R_6 . Then, we have

$$R_7 = \frac{1}{\frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_6}} = \frac{1}{\frac{1}{30} + \frac{1}{60} + \frac{1}{80}} \text{ k} = 16 \text{ k}\Omega$$

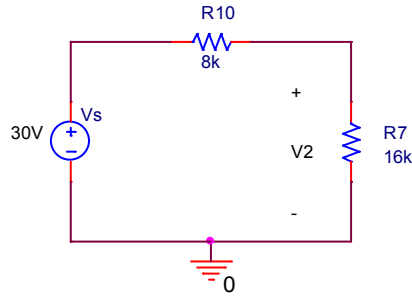
Let R_8 be the equivalent resistance of the series connection of R_1 and R_2 . Then, we have

$$R_8 = R_1 + R_2 = 10 \text{ k}\Omega + 30 \text{ k}\Omega = 40 \text{ k}\Omega.$$

Let R_{10} be the equivalent resistance of the parallel connection of R_3 and R_9 . Then, we have

$$R_{10} = \frac{R_3 R_9}{R_3 + R_9} = \frac{10 \times 40}{10 + 40} \text{ k} = \frac{400}{50} \text{ k} = 8 \text{ k}\Omega$$

R_{10} is in series with R_7 . The circuit reduces to



The voltage V_2 across R_7 is given by

$$V_2 = \frac{R_7}{R_{10} + R_7} V_S = \frac{16}{8 + 16} \times 30 \text{ V} = \frac{16}{24} \times 30 \text{ V} = 20 \text{ V}$$

The voltage across R_{10} is given by

$$V_{R_{10}} = \frac{R_{10}}{R_{10} + R_7} V_S = \frac{8}{8 + 16} \times 30 \text{ V} = \frac{8}{24} \times 30 \text{ V} = 10 \text{ V}$$

The voltage $V_{R_{10}}$ is split between R_1 and R_2 in proportion to the resistance values. Applying the voltage divider rule, we obtain

$$V_1 = V_2 + \frac{R_2}{R_1 + R_2} V_{R_{10}} = 20 + \frac{30}{10 + 30} \times 10 \text{ V} = 20 + \frac{30}{40} \times 10 \text{ V} = 27.5 \text{ V}$$

Problem 2.56

Let R_7 be the equivalent resistance of the parallel connection of $R_2 + R_4$ and $R_3 + R_5$. Then we have

$$R_7 = (R_2 + R_4) \parallel (R_3 + R_5) = 5k \parallel 5k = \frac{5k \times 5k}{5k + 5k} = \frac{25k^2}{10k} = 2.5 k\Omega$$

The voltage V_S is divided across R_1 , R_7 , and R_6 in proportion to the resistance values. The voltage across R_7 is given by

$$V_{R_7} = \frac{R_7}{R_1 + R_7 + R_6} V_S = \frac{2.5}{1 + 2.5 + 1.5} \times 10 \text{ V} = \frac{2.5}{5} \times 10 \text{ V} = 5 \text{ V}$$

The voltage across R_1 is given by

$$V_{R_1} = \frac{R_1}{R_1 + R_7 + R_6} V_S = \frac{1}{1 + 2.5 + 1.5} \times 10 \text{ V} = \frac{1}{5} \times 10 \text{ V} = 2 \text{ V}$$

The voltage across R_6 is given by

$$V_{R6} = \frac{R_6}{R_1 + R_7 + R_6} V_S = \frac{1.5}{1 + 2.5 + 1.5} \times 10V = \frac{1.5}{5} \times 10V = 3V$$

The voltage V_{R7} is divided across R_2 and R_4 in proportion to the resistance values. Thus, we have

$$V_{R2} = \frac{R_2}{R_2 + R_4} V_{R7} = \frac{1}{1 + 4} \times 5V = \frac{1}{5} \times 5V = 1V$$

$$V_{R4} = \frac{R_4}{R_2 + R_4} V_{R7} = \frac{4}{1 + 4} \times 5V = \frac{4}{5} \times 5V = 4V$$

The voltage V_{R7} is divided across R_3 and R_5 in proportion to the resistance values. Thus, we have

$$V_{R3} = \frac{R_3}{R_3 + R_5} V_{R7} = \frac{3}{3 + 2} \times 5V = \frac{3}{5} \times 5V = 3V$$

$$V_{R5} = \frac{R_5}{R_3 + R_5} V_{R7} = \frac{2}{3 + 2} \times 5V = \frac{2}{5} \times 5V = 2V$$

The voltage at node a , V_a , is the sum of V_{R4} and V_{R6} . Thus, we have

$$V_a = V_{R4} + V_{R6} = 4V + 3V = 7V$$

The voltage at node b , V_b , is the sum of V_{R5} and V_{R6} . Thus, we have

$$V_b = V_{R5} + V_{R6} = 2V + 3V = 5V$$

The voltage V_{ab} is the difference of V_a and V_b , that is,

$$V_{ab} = V_a - V_b = 7V - 5V = 2V.$$

Problem 2.57

Let $R_7 = R_2 \parallel R_3$ and $R_8 = R_5 \parallel R_6$. Then, we have

$$R_7 = R_2 \parallel R_3 = \frac{R_2 \times R_3}{R_2 + R_3} = \frac{5k\Omega \times 5k\Omega}{5k\Omega + 5k\Omega} = \frac{25}{10} k\Omega = 2.5k\Omega$$

$$R_8 = R_5 \parallel R_6 = \frac{R_5 \times R_6}{R_5 + R_6} = \frac{2k\Omega \times 8k\Omega}{2k\Omega + 8k\Omega} = \frac{16}{10} k\Omega = 1.6k\Omega$$

The equivalent resistance seen from the voltage source is

$$R_{eq} = R_1 + R_7 + R_4 + R_8 = 0.5 \text{ k}\Omega + 2.5 \text{ k}\Omega + 0.4 \text{ k}\Omega + 1.6 \text{ k}\Omega = 5 \text{ k}\Omega$$

From Ohm's law, the current I_1 is given by

$$I_1 = \frac{V_S}{R_{eq}} = \frac{10V}{5k\Omega} = 2mA$$

The voltage drop across R_1 is $I_1R_1 = 2mA \times 0.5k\Omega = 1V$. The voltage V_1 is given by

$$V_1 = V_S - I_1R_1 = 10 \text{ V} - 1V = 9V.$$

Since $R_2 = R_3$, $I_2 = I_3 = I_1/2 = 1mA$. The voltage drop across R_7 is $I_1 \times R_7 = 2mA \times 2.5k\Omega = 5V$. We can get the same voltage drop from $I_2R_2 = I_3R_3 = 5V$. The voltage V_2 is given by

$$V_2 = V_1 - 5V = 9V - 5V = 4V.$$

The voltage drop across R_4 is $I_1 \times R_4 = 2mA \times 0.4k\Omega = 0.8V$. The voltage V_3 is given by

$$V_3 = V_2 - 0.8V = 4V - 0.8V = 3.2V.$$

The current through R_5 is given by

$$I_4 = \frac{V_3}{R_5} = \frac{3.2V}{2k\Omega} = 1.6mA$$

The current through R_6 is given by

$$I_5 = \frac{V_3}{R_6} = \frac{3.2V}{8k\Omega} = 0.4mA$$

Problem 2.58

From the current divider rule, the current I_{R1} is given by

$$I_{R1} = \frac{R_2}{R_1 + R_2} I_S = \frac{3}{2+3} \times 10mA = 6mA$$

Similarly, the current I_{R2} is given by

$$I_{R2} = \frac{R_1}{R_1 + R_2} I_S = \frac{2}{2+3} \times 10mA = 4mA$$

Problem 2.59

From the current divider rule, the current I_{R1} is given by

$$I_{R1} = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} I_S = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}} \times 26 \text{ mA} = \frac{\frac{1}{2}}{\frac{6}{12} + \frac{4}{12} + \frac{3}{12}} \times 26 \text{ mA} = 12 \text{ mA}$$

Similarly, the currents I_{R2} and I_{R3} are given respectively by

$$I_{R2} = \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} I_S = \frac{\frac{1}{3}}{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}} \times 26 \text{ mA} = \frac{\frac{1}{3}}{\frac{6}{12} + \frac{4}{12} + \frac{3}{12}} \times 26 \text{ mA} = 8 \text{ mA}$$

$$I_{R3} = \frac{\frac{1}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} I_S = \frac{\frac{1}{4}}{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}} \times 26 \text{ mA} = \frac{\frac{1}{4}}{\frac{6}{12} + \frac{4}{12} + \frac{3}{12}} \times 26 \text{ mA} = 6 \text{ mA}$$

Problem 2.60

Let R_6 be the equivalent resistance of the parallel connection of R_2 and R_3 . Then, R_6 is given by

$$R_6 = \frac{R_2 R_3}{R_2 + R_3} = \frac{30 \text{ k} \times 60 \text{ k}}{30 \text{ k} + 60 \text{ k}} = \frac{1800}{90} \text{ k} = 20 \text{ k}\Omega$$

Let R_7 be the equivalent resistance of the parallel connection of R_4 and R_5 . Then, R_7 is given by

$$R_7 = \frac{R_4 R_5}{R_4 + R_5} = \frac{90 \text{ k} \times 180 \text{ k}}{90 \text{ k} + 180 \text{ k}} = \frac{180}{3} \text{ k} = 60 \text{ k}\Omega$$

Let R_8 be the equivalent resistance of the series connection of R_6 and R_7 . Then, R_8 is given by

$$R_8 = R_6 + R_7 = 80 \text{ k}\Omega$$

The current from the current source I_S is split into I_{R1} and I_{R8} according to the current divider rule. Thus, we have

$$I_{R1} = \frac{R_8}{R_1 + R_8} I_S = \frac{80}{20 + 80} \times 48 \text{ mA} = 38.4 \text{ mA}$$

$$I_{R8} = \frac{R_1}{R_1 + R_8} I_S = \frac{20}{20 + 80} \times 48 \text{ mA} = 9.6 \text{ mA}$$

The current I_{R8} is split into I_{R2} and I_{R3} according to the current divider rule. Thus, we have

$$I_{R2} = \frac{R_3}{R_2 + R_3} I_{R8} = \frac{60}{30 + 60} \times 9.6 \text{ mA} = 6.4 \text{ mA}$$

$$I_{R3} = \frac{R_2}{R_2 + R_3} I_{R8} = \frac{30}{30 + 60} \times 9.6 \text{ mA} = 3.2 \text{ mA}$$

The current I_{R8} is split into I_{R4} and I_{R5} according to the current divider rule. Thus, we have

$$I_{R4} = \frac{R_5}{R_4 + R_5} I_{R8} = \frac{180}{90 + 180} \times 9.6 \text{ mA} = 6.4 \text{ mA}$$

$$I_{R5} = \frac{R_4}{R_4 + R_5} I_{R8} = \frac{90}{90 + 180} \times 9.6 \text{ mA} = 3.2 \text{ mA}$$

Problem 2.61

$$R_3 \parallel R_4 = \frac{R_3 \times R_4}{R_3 + R_4} = \frac{4 \text{ k}\Omega \times 6 \text{ k}\Omega}{4 \text{ k}\Omega + 6 \text{ k}\Omega} = \frac{24}{10} \text{ k}\Omega = 2.4 \text{ k}\Omega$$

$$R_5 = R_2 + (R_3 \parallel R_4) = 0.6 \text{ k}\Omega + 2.4 \text{ k}\Omega = 3 \text{ k}\Omega$$

The current from the current source, $I_S = 2 \text{ mA}$, is split between I_1 and I_2 based on the current divider rule.

$$I_1 = I_S \times \frac{R_5}{R_1 + R_5} = 2 \text{ mA} \times \frac{3 \text{ k}\Omega}{7 \text{ k}\Omega + 3 \text{ k}\Omega} = 0.6 \text{ mA}$$

$$I_2 = I_S \times \frac{R_1}{R_1 + R_5} = 2 \text{ mA} \times \frac{7 \text{ k}\Omega}{7 \text{ k}\Omega + 3 \text{ k}\Omega} = 1.4 \text{ mA}$$

The currents I_3 and I_4 are found by applying the current divider rule on R_3 and R_4 .

$$I_3 = I_2 \times \frac{R_4}{R_3 + R_4} = 1.4 \text{ mA} \times \frac{6 \text{ k}\Omega}{4 \text{ k}\Omega + 6 \text{ k}\Omega} = 0.84 \text{ mA}$$

$$I_4 = I_2 \times \frac{R_3}{R_3 + R_4} = 1.4 \text{ mA} \times \frac{4 \text{ k}\Omega}{4 \text{ k}\Omega + 6 \text{ k}\Omega} = 0.56 \text{ mA}$$

The voltages V_1 and V_2 are found by applying Ohm's law.

$$V_1 = I_1 \times R_1 = 0.6 \text{ mA} \times 7 \text{ k}\Omega = 4.2 \text{ V}$$

$$V_2 = I_3 \times R_3 = 0.84 \text{ mA} \times 4 \text{ k}\Omega = 3.36 \text{ V}$$

Problem 2.62

Let R_a be the equivalent resistance of the series connection of R_2 and R_3 . Then, we have

$$R_a = R_2 + R_3 = 2 \text{ k}\Omega + 5 \text{ k}\Omega = 7 \text{ k}\Omega$$

Application of current divider rule yields

$$I_1 = I_s \times \frac{R_a}{R_1 + R_a} = 20 \text{ mA} \times \frac{7}{3 + 7} = 14 \text{ mA}$$

$$I_2 = I_s \times \frac{R_1}{R_1 + R_a} = 20 \text{ mA} \times \frac{3}{3 + 7} = 6 \text{ mA}$$

Problem 2.63

Let R_a be the equivalent resistance of the parallel connection of R_2 and R_3 . Then, we have

$$R_a = \frac{R_2 \times R_3}{R_2 + R_3} = \frac{20 \times 20}{20 + 20} \text{ k} = 10 \text{ k}\Omega$$

Application of voltage divider rule yields

$$V_1 = V_s \times \frac{R_a}{R_1 + R_a} = 50 \text{ V} \times \frac{10}{15 + 10} = 20 \text{ V}$$

Application of Ohm's law yields

$$I_2 = \frac{V_1}{R_2} = \frac{20 \text{ V}}{20 \text{ k}\Omega} = 1 \text{ mA}$$

$$I_3 = \frac{V_1}{R_3} = \frac{20 \text{ V}}{20 \text{ k}\Omega} = 1 \text{ mA}$$

From KCL, we have

$$I_1 = I_2 + I_3 = 1 \text{ mA} + 1 \text{ mA} = 2 \text{ mA}$$

Problem 2.64

Let R_8 be the equivalent resistance of the parallel connection of R_6 and R_7 . Then, R_8 is given by

$$R_8 = \frac{R_6 R_7}{R_6 + R_7} = \frac{9k \times 18k}{9k + 18k} = \frac{18}{3} k = 6 \text{ k}\Omega$$

Let R_9 be the equivalent resistance of the series connection of R_5 and R_8 . Then, R_9 is given by

$$R_9 = R_5 + R_8 = 10 \text{ k}\Omega$$

Let R_{10} be the equivalent resistance of the parallel connection of R_3 , R_4 and R_9 . Then, R_{10} is given by

$$R_{10} = \frac{1}{\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_9}} = \frac{1}{\frac{1}{20k} + \frac{1}{20k} + \frac{1}{10k}} = \frac{20k}{4} = 5 \text{ k}\Omega$$

Let R_{11} be the equivalent resistance of the series connection of R_2 and R_{10} . Then, R_{11} is given by

$$R_{11} = R_2 + R_{10} = 10 \text{ k}\Omega$$

The current from the current source I_S is split into I_{R1} and I_{R11} according to the current divider rule. Thus, we have

$$I_{R1} = \frac{R_{11}}{R_1 + R_{11}} I_S = \frac{10}{15 + 10} \times 50 \text{ mA} = 20 \text{ mA}$$

$$I_{R11} = \frac{R_1}{R_1 + R_{11}} I_S = \frac{15}{15 + 10} \times 50 \text{ mA} = 30 \text{ mA}$$

Notice that $I_{R2} = I_{R11} = 30 \text{ mA}$.

The current I_{R11} is split into I_{R3} , I_{R4} and I_{R9} according to the current divider rule. Thus, we have

$$I_{R3} = \frac{\frac{1}{R_3}}{\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_9}} I_{R11} = \frac{\frac{1}{20}}{\frac{1}{20} + \frac{1}{20} + \frac{1}{10}} \times 30 \text{ mA} = \frac{1}{4} \times 30 \text{ mA} = 7.5 \text{ mA}$$

$$I_{R4} = \frac{\frac{1}{R_4}}{\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_9}} I_{R11} = \frac{\frac{1}{20}}{\frac{1}{20} + \frac{1}{20} + \frac{1}{10}} \times 30 \text{ mA} = \frac{1}{4} \times 30 \text{ mA} = 7.5 \text{ mA}$$

$$I_{R9} = \frac{\frac{1}{R_9}}{\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_9}} I_{R11} = \frac{\frac{1}{10k}}{\frac{1}{20k} + \frac{1}{20k} + \frac{1}{10k}} \times 30 \text{ mA} = \frac{2}{4} \times 30 \text{ mA} = 15 \text{ mA}$$

Notice that $I_{R5} = I_{R9} = 15 \text{ mA}$.

The current I_{R9} is split into I_{R6} and I_{R7} according to the current divider rule. Thus, we have

$$I_{R6} = \frac{R_7}{R_6 + R_7} I_{R9} = \frac{18k}{9k + 18k} \times 15 \text{ mA} = \frac{2}{3} \times 15 \text{ mA} = 10 \text{ mA}$$

$$I_{R7} = \frac{R_6}{R_6 + R_7} I_{R9} = \frac{9k}{9k + 18k} \times 15 \text{ mA} = \frac{1}{3} \times 15 \text{ mA} = 5 \text{ mA}$$

Problem 2.65

Let R_a be the equivalent resistance of the parallel connection of R_1 and R_2 . Then, we have

$$R_a = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{90 \times 180}{90 + 180} = \frac{1 \times 180}{1 + 2} = 60 \Omega$$

Let R_b be the equivalent resistance of the parallel connection of R_4 and R_5 . Then, we have

$$R_b = \frac{R_4 \times R_5}{R_4 + R_5} = \frac{100 \times 150}{100 + 150} = \frac{2 \times 150}{2 + 3} = 60 \Omega$$

Let R_c be the equivalent resistance of the series connection of R_a and R_b . Then, we have

$$R_c = R_a + R_b = 60 \Omega + 60 \Omega = 120 \Omega$$

Application of current divider rule yields

$$I_3 = I_s \times \frac{R_c}{R_3 + R_c} = 9.6 \text{ mA} \times \frac{120}{360 + 120} = 2.4 \text{ mA}$$

From Ohm's law, the voltage across R_3 is given by

$$V_1 = R_3 I_3 = 360 \, \Omega \times 0.0024 \, \text{A} = 0.864 \, \text{V}$$

Application of voltage divider rule yields

$$V_2 = V_1 \times \frac{R_b}{R_a + R_b} = 0.864 \, \text{V} \times \frac{60}{60 + 60} = 0.432 \, \text{V}$$

Application of Ohm's law yields

$$I_1 = \frac{V_1 - V_2}{R_1} = \frac{0.864 - 0.432}{90} = \frac{0.432}{90} = 4.8 \, \text{mA}$$

$$I_2 = \frac{V_1 - V_2}{R_2} = \frac{0.864 - 0.432}{180} = \frac{0.432}{180} = 2.4 \, \text{mA}$$

$$I_4 = \frac{V_2}{R_4} = \frac{0.432}{100} = 4.32 \, \text{mA}$$

$$I_5 = \frac{V_2}{R_5} = \frac{0.432}{150} = 2.88 \, \text{mA}$$

MATLAB

```
clear all; format long;
R1=90;R2=180;R3=360;R4=100;R5=150;
Is=9.6e-3;
Ra=P([R1,R2])
Rb=P([R4,R5])
Rc=Ra+Rb
I3=Is*Rc/(R3+Rc)
V1=R3*I3
V2=V1*Rb/(Ra+Rb)
I1=(V1-V2)/R1
I2=(V1-V2)/R2
I4=V2/R4
I5=V2/R5
```

Answers:

```
Ra =
    60
Rb =
    60
Rc =
   120
I3 =
  0.0024000000000000
V1 =
  0.8640000000000000
V2 =
  0.4320000000000000
I1 =
  0.0048000000000000
I2 =
  0.0024000000000000
I4 =
```

$$I_5 = \frac{0.004320000000000}{0.002880000000000}$$

Problem 2.66

Let R_a be the equivalent resistance of the series connection of R_5 and R_6 . Then, we have

$$R_a = R_5 + R_6 = 10 \Omega + 5 \Omega = 15 \Omega$$

Let R_b be the equivalent resistance of the parallel connection of R_4 and R_a . Then, we have

$$R_b = \frac{R_4 \times R_a}{R_4 + R_a} = \frac{10 \times 15}{10 + 15} = \frac{150}{25} = 6 \Omega$$

Let R_c be the equivalent resistance of the series connection of R_3 and R_b . Then, we have

$$R_c = R_3 + R_b = 10 \Omega + 6 \Omega = 16 \Omega$$

Let R_d be the equivalent resistance of the parallel connection of R_2 and R_c . Then, we have

$$R_d = \frac{R_2 \times R_c}{R_2 + R_c} = \frac{20 \times 16}{20 + 16} = \frac{320}{36} = \frac{80}{9} = 8.8889 \Omega$$

Let R_e be the equivalent resistance of the series connection of R_1 and R_d . Then, we have

$$R_e = R_1 + R_d = 4 \Omega + 8.8889 \Omega = 12.8889 \Omega$$

Application of Ohm's law yields

$$I_1 = \frac{V_s}{R_e} = \frac{100}{12.8889} = 7.9786 \text{ A}$$

$$V_1 = R_1 I_1 = 4 \times 7.9786 = 31.0345 \text{ V}$$

From KVL, we have

$$V_2 = V_s - V_1 = 100 - 31.0345 = 68.9655 \text{ V}$$

Application of Ohm's law yields

$$I_2 = \frac{V_2}{R_2} = \frac{68.9655 \text{ V}}{20 \Omega} = 3.4483 \text{ A}$$

From KCL, we have

$$I_3 = I_1 - I_2 = 7.9786 - 3.4483 = 4.3103 \text{ A}$$

From Ohm's law, we have

$$V_3 = R_3 I_3 = 10 \times 4.3103 = 43.1034 \text{ V}$$

From KVL, we have

$$V_4 = V_2 - V_3 = 68.9655 - 43.1034 = 25.8621 \text{ V}$$

Application of Ohm's law yields

$$I_4 = \frac{V_4}{R_4} = \frac{25.8621 \text{ V}}{10 \Omega} = 2.5862 \text{ A}$$

$$I_5 = \frac{V_4}{R_a} = \frac{25.8621 \text{ V}}{15 \Omega} = 1.7241 \text{ A}$$

$$V_5 = R_5 I_5 = 10 \times 1.7241 = 17.2414 \text{ V}$$

$$V_6 = R_6 I_5 = 5 \times 1.7241 = 8.6207 \text{ V}$$

MATLAB

```
clear all; format long;
R1=4;R2=20;R3=10;R4=10;R5=10;R6=5;
Vs=100;
Ra=R5+R6
Rb=P([R4,Ra])
Rc=R3+Rb
Rd=P([R2,Rc])
Re=R1+Rd
I1=Vs/Re
V1=R1*I1
V2=Vs-V1
I2=V2/R2
I3=I1-I2
V3=R3*I3
V4=V2-V3
I4=V4/R4
I5=V4/Ra
V5=R5*I5
V6=R6*I5
SV=-Vs+V1+V3+V5+V6
SI=-I1+I2+I4+I5
```

Answers:

```
Ra =
    15
Rb =
  5.999999999999999
Rc =
    16
Rd =
  8.888888888888889
Re =
  12.888888888888889
```

```

I1 =
  7.758620689655173
V1 =
  31.034482758620690
V2 =
  68.965517241379303
I2 =
  3.448275862068965
I3 =
  4.310344827586207
V3 =
  43.103448275862071
V4 =
  25.862068965517231
I4 =
  2.586206896551723
I5 =
  1.724137931034482
V5 =
  17.241379310344819
V6 =
  8.620689655172409
SV =
  -3.552713678800501e-15
SI =
  -1.998401444325282e-15

```

Problem 2.67

Let R_a be the equivalent resistance of the parallel connection of $R_6 = 4 \Omega$ and $R_7 + R_8 + R_9 = 12 \Omega$. Then, we have

$$R_a = \frac{4 \times 12}{4 + 12} = \frac{48}{16} = 3 \Omega$$

Let R_b be the equivalent resistance of the parallel connection of $R_2 = 4 \Omega$ and $R_3 + R_4 + R_5 = 12 \Omega$. Then, we have

$$R_b = \frac{4 \times 12}{4 + 12} = \frac{48}{16} = 3 \Omega$$

Let R_c be the equivalent resistance of the series connection of R_1 , R_a , and R_b . Then, we have

$$R_c = R_1 + R_a + R_b = 4 \Omega + 3 \Omega + 3 \Omega = 10 \Omega$$

The current through R_1 is

$$I_1 = \frac{V_2}{R_c} = \frac{40V}{10\Omega} = 4A$$

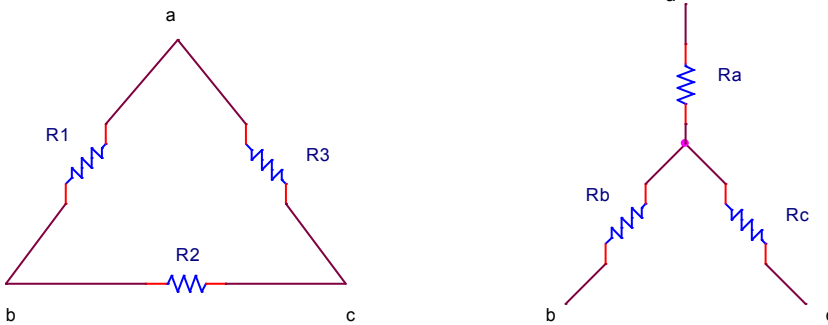
Application of current divider rule yields

$$I = 4A \times \frac{4}{4 + 12} = \frac{16}{16} A = 1A$$

Problem 2.68

Resistors R_1 , R_2 , and R_3 are connected in delta. These three resistors can be transformed to wye configuration with resistors R_a , R_b , and R_c using

$$R_a = \frac{R_1 R_3}{R_1 + R_2 + R_3}, R_b = \frac{R_1 R_2}{R_1 + R_2 + R_3}, R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$



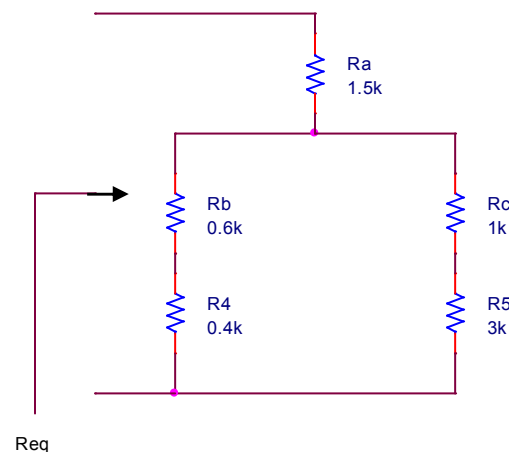
Substituting the values, we obtain

$$R_a = \frac{R_1 R_3}{R_1 + R_2 + R_3} = \frac{3 \times 5}{3 + 2 + 5} = \frac{15}{10} = 1.5 \text{ k}\Omega$$

$$R_b = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{3 \times 2}{3 + 2 + 5} = \frac{6}{10} = 0.6 \text{ k}\Omega$$

$$R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3} = \frac{2 \times 5}{3 + 2 + 5} = \frac{10}{10} = 1 \text{ k}\Omega$$

The circuit shown in Figure P2.68 can be redrawn as that shown below.



The sum of R_b and R_4 is $1\text{ k}\Omega$, and the sum of R_c and R_5 is $4\text{ k}\Omega$. These two are connected in parallel. Thus, we have

$$(R_b + R_4) \parallel (R_c + R_5) = 1 \parallel 4 = \frac{1 \times 4}{1 + 4} = \frac{4}{5} = 0.8\text{ k}\Omega$$

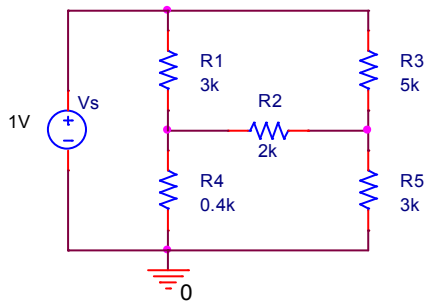
The equivalent resistance R_{eq} is the sum of R_a and $(R_b + R_4) \parallel (R_c + R_5)$:

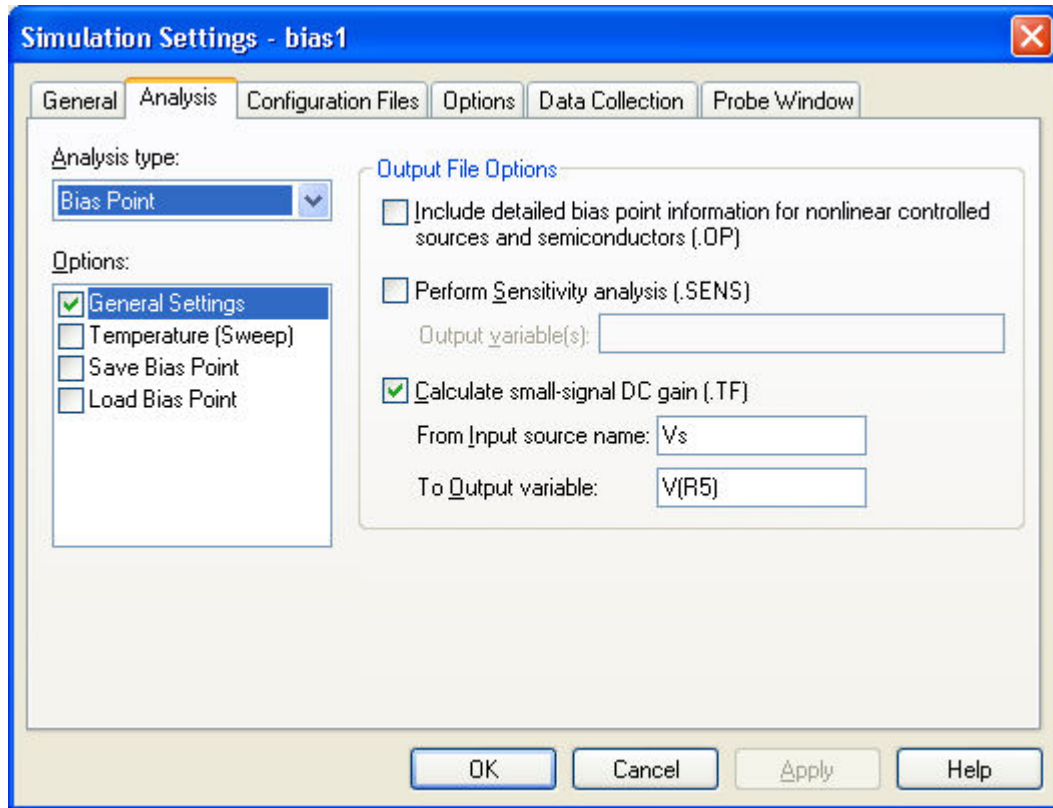
$$R_{eq} = R_a + 0.8 = 1.5 + 0.8 = 2.3\text{ k}\Omega.$$

MATLAB

```
>> [Ra, Rb, Rc]=D2Y ([3000, 2000, 5000])
Ra =
    1500
Rb =
     600
Rc =
    1000
>> Req=Ra+P ([Rb+400, Rc+3000])
Req =
    2300
```

PSpice





Click on View Simulation Output File. Part of the output file reads

```

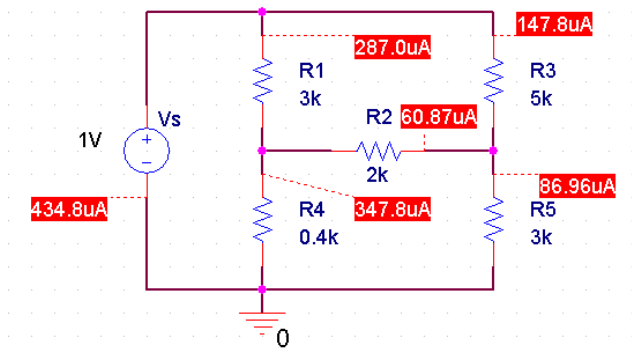
****      SMALL-SIGNAL CHARACTERISTICS

V(R_R5) / V_Vs =  2.609E-01

INPUT RESISTANCE AT V_Vs =  2.300E+03

OUTPUT RESISTANCE AT V(R_R5) =  1.043E+03
    
```

The input resistance is 2.3 kΩ. Alternatively, just run the bias point analysis (uncheck .TF) and display currents.



The current through the voltage source is $434.8\mu\text{A}$. The input resistance is given by the ratio of the test voltage 1V to the current. Thus, we have

$$R_{eq} = \frac{1\text{V}}{434.8 \times 10^{-6}} = 2.2999\text{ k}\Omega$$

Problem 2.69

The wye-connected resistors R_a , R_b , and R_c can be transformed to delta connected resistors R_1 , R_2 , and R_3 .

$$R_1 = \frac{R_a R_b + R_b R_c + R_a R_c}{R_c} = \frac{14.4 \times 21.6 + 21.6 \times 12.96 + 14.4 \times 12.96}{12.96} = 60\text{ k}\Omega$$

$$R_2 = \frac{R_a R_b + R_b R_c + R_a R_c}{R_a} = \frac{14.4 \times 21.6 + 21.6 \times 12.96 + 14.4 \times 12.96}{14.4} = 54\text{ k}\Omega$$

$$R_3 = \frac{R_a R_b + R_b R_c + R_a R_c}{R_b} = \frac{14.4 \times 21.6 + 21.6 \times 12.96 + 14.4 \times 12.96}{21.6} = 36\text{ k}\Omega$$

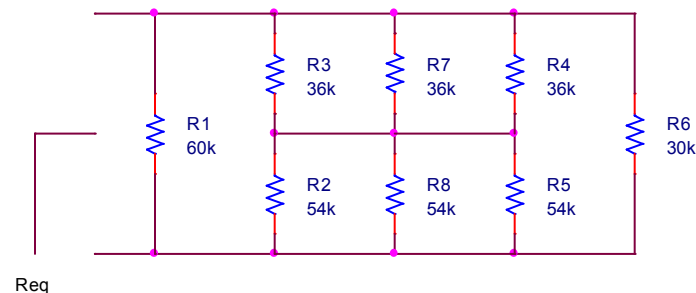
Similarly, the wye-connected resistors R_d , R_e , and R_f can be transformed to delta connected resistors R_4 , R_5 , and R_6 .

$$R_4 = \frac{R_d R_e + R_e R_f + R_d R_f}{R_f} = \frac{9 \times 16.2 + 16.2 \times 13.5 + 9 \times 13.5}{13.5} = 36\text{ k}\Omega$$

$$R_5 = \frac{R_d R_e + R_e R_f + R_d R_f}{R_d} = \frac{9 \times 16.2 + 16.2 \times 13.5 + 9 \times 13.5}{9} = 54\text{ k}\Omega$$

$$R_6 = \frac{R_d R_e + R_e R_f + R_d R_f}{R_e} = \frac{9 \times 16.2 + 16.2 \times 13.5 + 9 \times 13.5}{16.2} = 30\text{ k}\Omega$$

After two wye-delta transformations, the circuit shown in Figure P2.69 is transformed to the circuit shown below.



The equivalent resistance of the parallel connection of R_3 , R_7 , and R_4 is given by

$$R_g = \frac{1}{\frac{1}{R_3} + \frac{1}{R_7} + \frac{1}{R_7}} = \frac{1}{\frac{1}{36} + \frac{1}{36} + \frac{1}{36}} = \frac{1}{\frac{3}{36}} = \frac{36}{3} = 12 \text{ k}\Omega$$

The equivalent resistance of the parallel connection of R_2 , R_8 , and R_5 is given by

$$R_h = \frac{1}{\frac{1}{R_2} + \frac{1}{R_8} + \frac{1}{R_5}} = \frac{1}{\frac{1}{54} + \frac{1}{54} + \frac{1}{54}} = \frac{1}{\frac{3}{54}} = \frac{54}{3} = 18 \text{ k}\Omega$$

Resistors R_g and R_h are connected in series. The equivalent resistance of R_g and R_h is given by

$$R_i = R_g + R_h = 12 + 18 = 30 \text{ k}\Omega.$$

The equivalent resistance R_{eq} of the circuit shown in Figure P2.10 is given by the parallel connection of R_1 , R_i , and R_6 , that is,

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_i} + \frac{1}{R_6}} = \frac{1}{\frac{1}{60} + \frac{1}{30} + \frac{1}{30}} = \frac{1}{\frac{5}{60}} = \frac{60}{5} = 12 \text{ k}\Omega$$

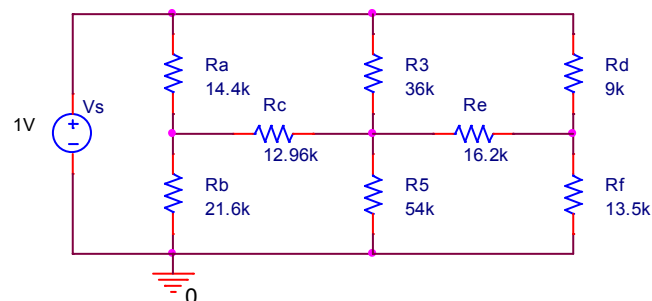
MATLAB

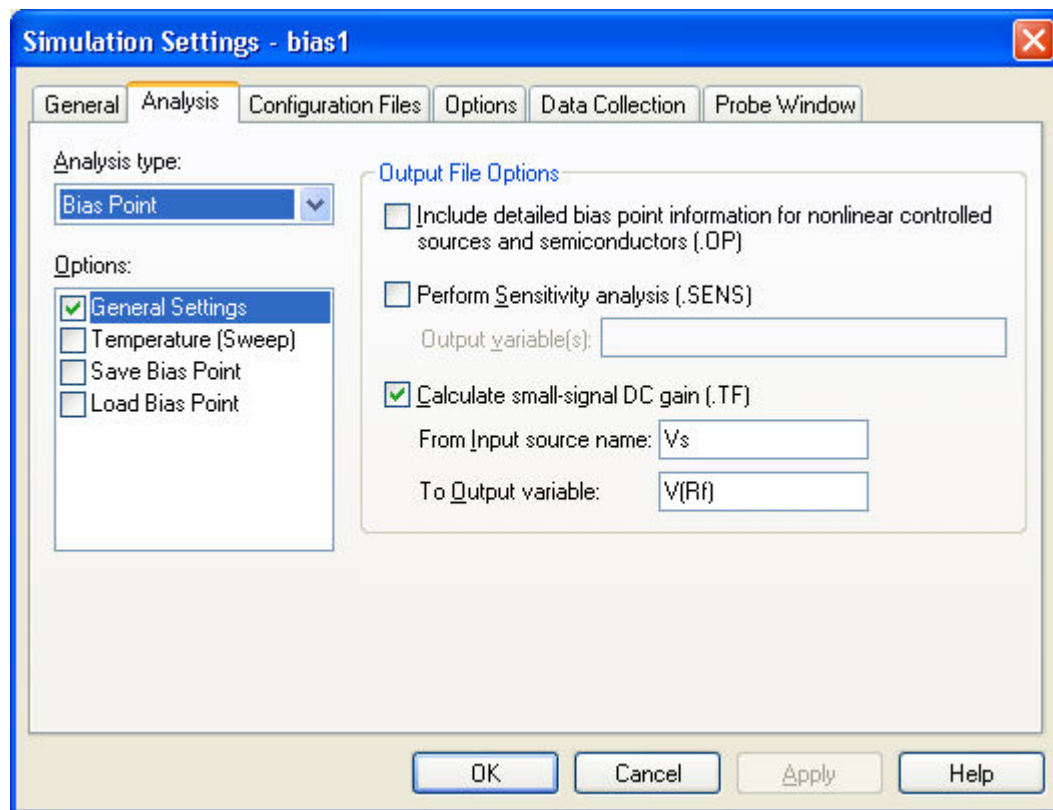
```
clear all;
Ra=14400;Rb=21600;Rc=12960;Rd=9000;Re=16200;Rf=13500;R7=36000;R8=54000;
[R1,R2,R3]=Y2D([Ra,Rb,Rc])
[R4,R5,R6]=Y2D([Rd,Re,Rf])
Req=P([R1,R6,P([R3,R7,R4])+P([R2,R8,R5])])
```

Answer:

```
Req =
    1.2000e+04
```

PSpice





View Simulation Output File.

```

****      SMALL-SIGNAL CHARACTERISTICS

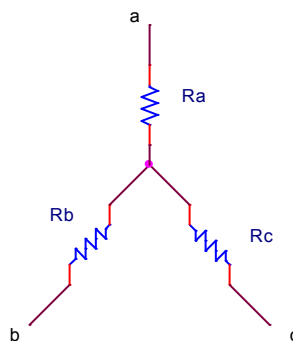
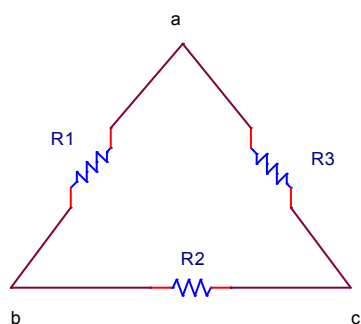
V(R_Rf)/V_Vs = 6.000E-01

INPUT RESISTANCE AT V_Vs = 1.200E+04

OUTPUT RESISTANCE AT V(R_Rf) = 4.500E+03
    
```

The input resistance is $R_{eq} = 12 \text{ k}\Omega$.

Problem 2.70



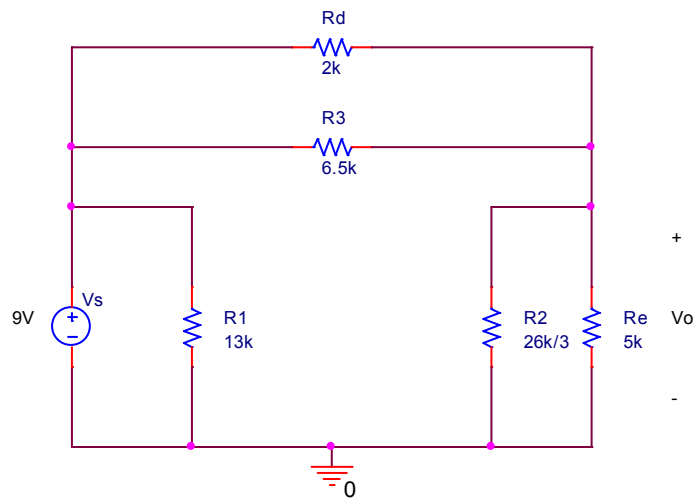
The wye-connected resistors R_a , R_b , and R_c can be transformed to delta connected resistors R_1 , R_2 , and R_3 .

$$R_1 = \frac{R_a R_b + R_b R_c + R_a R_c}{R_c} = \frac{3 \times 4 + 4 \times 2 + 3 \times 2}{2} k = \frac{26}{2} k = 13 k\Omega$$

$$R_2 = \frac{R_a R_b + R_b R_c + R_a R_c}{R_a} = \frac{3 \times 4 + 4 \times 2 + 3 \times 2}{3} k = \frac{26}{3} k = 8.6667 k\Omega$$

$$R_3 = \frac{R_a R_b + R_b R_c + R_a R_c}{R_b} = \frac{3 \times 4 + 4 \times 2 + 3 \times 2}{4} k = \frac{26}{4} k = 6.5 k\Omega$$

The circuit shown in Figure P2.70 can be redrawn as that shown below.



Notice that

$$R_3 \parallel R_d = 1.5294 k\Omega, R_2 \parallel R_e = 3.1707 k\Omega$$

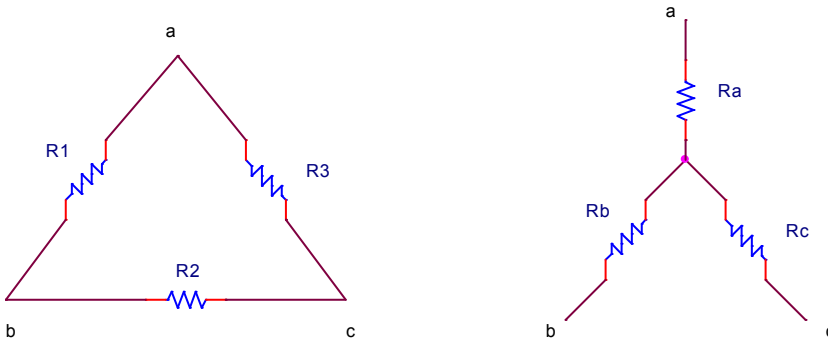
Application of voltage divider rule yields

$$V_o = V_s \times \frac{R_2 \parallel R_e}{R_3 \parallel R_d + R_2 \parallel R_e} = 9V \times \frac{3.1707}{1.5294 + 3.1707} = 6.0714V$$

Problem 2.71

Resistors R_1 , R_2 , and R_3 are connected in delta. These three resistors can be transformed to wye configuration with resistors R_a , R_b , and R_c using

$$R_a = \frac{R_1 R_3}{R_1 + R_2 + R_3}, R_b = \frac{R_1 R_2}{R_1 + R_2 + R_3}, R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$



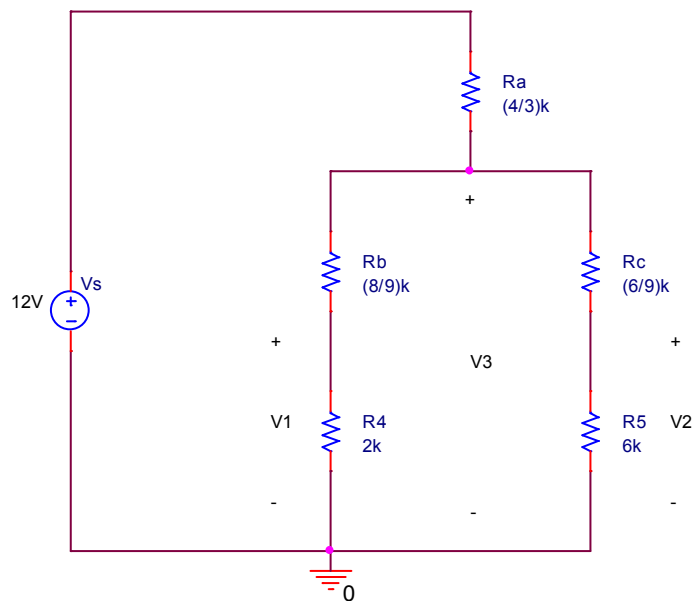
Substituting the values, we obtain

$$R_a = \frac{R_1 R_3}{R_1 + R_2 + R_3} = \frac{4 \times 3}{4 + 2 + 3} = \frac{12}{9} = \frac{4}{3} k\Omega = 1.3333 k\Omega$$

$$R_b = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{4 \times 2}{4 + 2 + 3} = \frac{8}{9} k\Omega = 0.8889 k\Omega$$

$$R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3} = \frac{2 \times 3}{4 + 2 + 3} = \frac{6}{9} k\Omega = 0.6667 k\Omega$$

The circuit shown in Figure P2.70 can be redrawn as that shown below.



Let

$$R_{10} = R_b + R_4 = 2.8889 \text{ k}\Omega$$

$$R_{11} = R_c + R_5 = 6.6667 \text{ k}\Omega$$

$$R_{12} = R_{10} \parallel R_{11} = \frac{R_{10} \times R_{11}}{R_{10} + R_{11}} = 2.0115 \text{ k}\Omega$$

V_3 = voltage across R_{10} and R_{11} .

Application of voltage divider rule yields

$$V_3 = V_s \times \frac{R_{12}}{R_a + R_{12}} = 9V \times \frac{2.0115}{1.3333 + 2.0115} = 7.2222V$$

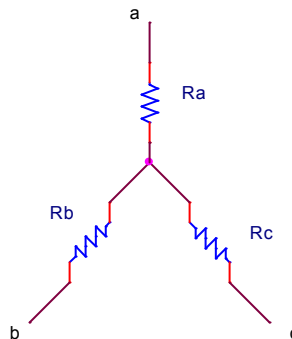
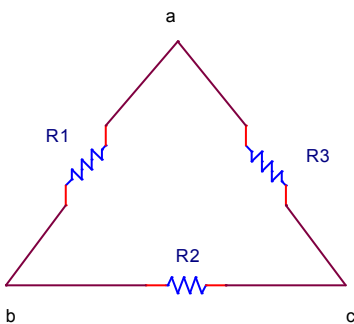
$$V_1 = V_3 \times \frac{R_4}{R_{10}} = 7.2222V \times \frac{2}{2.8889} = 5V$$

$$V_2 = V_3 \times \frac{R_5}{R_{11}} = 7.2222V \times \frac{6}{6.6667} = 6.5V$$

Problem 2.72

Resistors R_1 , R_2 , and R_3 are connected in delta. These three resistors can be transformed to wye configuration with resistors R_a , R_b , and R_c using

$$R_a = \frac{R_1 R_3}{R_1 + R_2 + R_3}, R_b = \frac{R_1 R_2}{R_1 + R_2 + R_3}, R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$



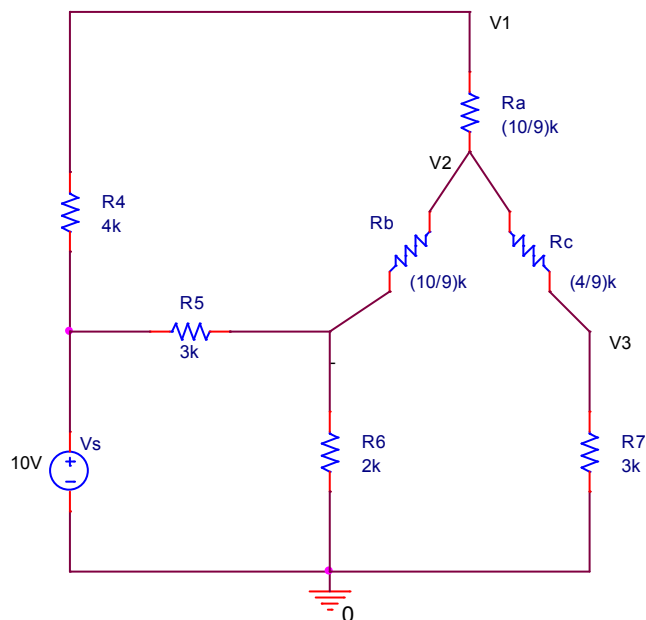
Substituting the values, we obtain

$$R_a = \frac{R_1 R_3}{R_1 + R_2 + R_3} = \frac{5 \times 2}{5 + 2 + 2} = \frac{10}{9} k\Omega = 1.1111 k\Omega$$

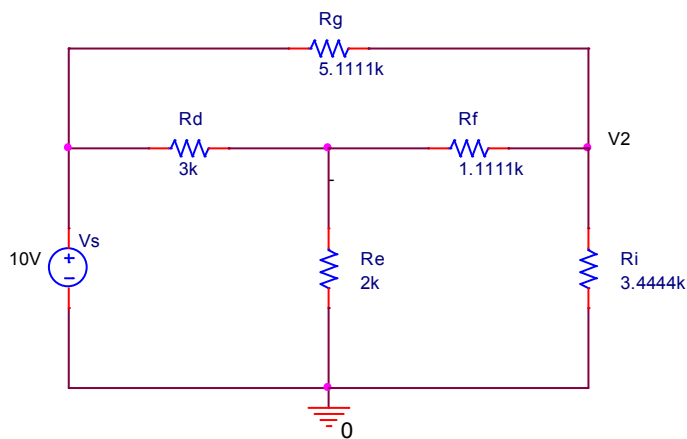
$$R_b = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{5 \times 2}{5 + 2 + 2} = \frac{10}{9} k\Omega = 1.1111 k\Omega$$

$$R_c = \frac{R_2 R_3}{R_1 + R_2 + R_3} = \frac{2 \times 2}{5 + 2 + 2} = \frac{4}{9} k\Omega = 0.4444 k\Omega$$

The circuit shown in Figure P2.70 can be redrawn as that shown below.



This circuit can be redrawn as



Notice that

$$R_g = R_4 + R_a = 5.1111 \text{ k}\Omega$$

$$R_d = R_5 = 3 \text{ k}\Omega$$

$$R_e = R_6 = 2 \text{ k}\Omega$$

$$R_f = R_b = 1.1111 \text{ k}\Omega$$

$$R_i = R_7 + R_c = 3.4444 \text{ k}\Omega$$

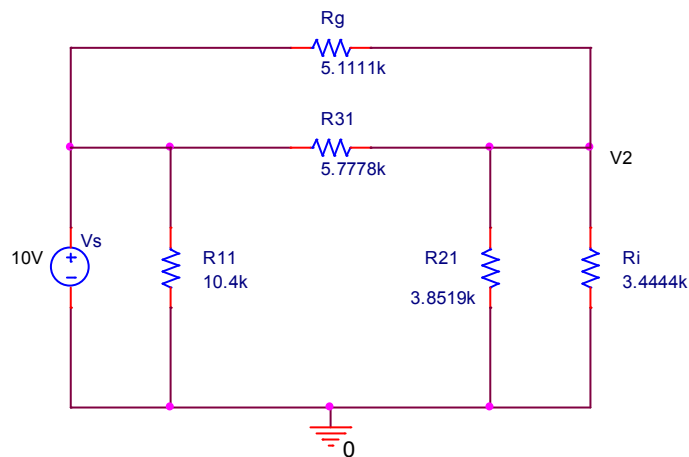
Converting the wye configuration R_d, R_e, R_f to delta configuration, we obtain

$$R_{11} = \frac{R_d R_e + R_e R_f + R_d R_f}{R_f} = \frac{3 \times 2 + 2 \times 1.1111 + 3 \times 1.1111}{1.1111} \text{ k} = 10.4 \text{ k}\Omega$$

$$R_{21} = \frac{R_d R_e + R_e R_f + R_d R_f}{R_d} = \frac{3 \times 2 + 2 \times 1.1111 + 3 \times 1.1111}{3} \text{ k} = 3.8519 \text{ k}\Omega$$

$$R_{31} = \frac{R_d R_e + R_e R_f + R_d R_f}{R_e} = \frac{3 \times 2 + 2 \times 1.1111 + 3 \times 1.1111}{2} \text{ k} = 5.7778 \text{ k}\Omega$$

The circuit with $R_{11}, R_{21},$ and R_{31} is shown below.



Let $R_{51} = R_g \parallel R_{31}$ and $R_{52} = R_i \parallel R_{21}$. Then, we have

$$R_{51} = \frac{R_g \times R_{31}}{R_g + R_{31}} = \frac{5.1111 \times 5.7778}{5.1111 + 5.7778} \text{ k} = 2.712 \text{ k}\Omega$$

$$R_{s2} = \frac{R_i \times R_{21}}{R_i + R_{21}} = \frac{3.4444 \times 3.8519}{3.4444 + 3.8519} k = 1.8184 k\Omega$$

Application of voltage divider rule yields

$$V_2 = V_s \times \frac{R_{s2}}{R_{s1} + R_{s2}} = 10V \times \frac{1.8184}{2.712 + 1.8184} = 4.0137V$$

Application of voltage divider rule yields

$$V_1 = V_2 + (V_s - V_2) \times \frac{R_a}{R_4 + R_a} = 4.0137V + 5.9863V \times \frac{1.1111}{5.1111} = 5.3151V$$

