CHAPTER TWO

Solutions for Section 2.1

Skill Refresher

S1. $5(x-3) = 5x - 15$.

S2. $a(2a+5) = 2a^2 + 5a$.

S3. $(m-5)(4(m-5)+2) = 4(m-5)^2+2(m-5) = 4(m^2-10m+25)+2m-10 = 4m^2-40m+100+2m-10 = 4m^2-38m+90$.

S4. $(x+2)(3x-8) = 3x^2 - 8x + 6x - 16 = 3x^2 - 2x - 16$.

S5.

$$\begin{aligned} 3\left(1+\frac{1}{x}\right) &= 3\left(\frac{x+1}{x}\right) \\ &= \frac{3x+3}{x}. \end{aligned}$$

S6. $3 + 2\left(\frac{1}{x}\right)^2 - x = 3 + \frac{2}{x^2} - x = \frac{3x^2+2-x^3}{x^2}$.

S7. $x^2 - 9 = (x-3)(x+3) = 0$. Hence $x = \pm 3$.

S8.

$$\begin{aligned} \sqrt{2x-1}+3 &= 9 \\ \sqrt{2x-1} &= 6 \\ 2x-1 &= 36 \\ 2x &= 37 \\ x &= \frac{37}{2}. \end{aligned}$$

S9.

$$\begin{aligned} \frac{21}{z-5} - \frac{13}{z^2-5z} &= 3 \\ \frac{21}{z-5} - \frac{13}{z(z-5)} &= 3 \\ \frac{21z-13}{z(z-5)} &= 3 \\ 21z - 13 &= 3z(z-5) \\ 21z - 13 &= 3z^2 - 15z \\ 3z^2 - 36z + 13 &= 0 \end{aligned}$$

$$\begin{aligned} z &= \frac{-(-36) \pm \sqrt{(-36)^2 - 4(3)(13)}}{2(3)} \\ &= \frac{36 \pm \sqrt{1140}}{6} \\ &= \frac{36 \pm \sqrt{4\cdot 285}}{6} \\ &= \frac{36 \pm 2\sqrt{285}}{6} \\ &= \frac{18 \pm \sqrt{285}}{3}. \end{aligned}$$

S10. Adding 1 to both sides, we get $2x^{3/2} = 8$. Dividing both sides by 2, we have $x^{3/2} = 4$. To solve it for x we raise both sides to the $2/3$ power to get

$$x = 4^{3/2}.$$

Exercises

1. Substituting -27 for x gives

$$g(-27) = -\frac{1}{2}(-27)^{1/3} = -\frac{1}{2}(-3) = \frac{3}{2}.$$

2. We need to solve for x in the equation:

$$0.3 = \frac{2x+1}{x+1}.$$

Multiplying both sides by $x + 1$ gives:

$$\begin{aligned} 0.3(x+1) &= 2x+1 \\ 0.3x + 0.3 &= 2x+1 \\ -1.7x &= 0.7 \\ x &= -0.412. \end{aligned}$$

3. **(a)** Substituting $t = 0$ gives $f(0) = 0^2 - 4 = -4$.
(b) Setting $f(t) = 0$ and solving gives $t^2 - 4 = 0$, so $t^2 = 4$, so $t = \pm 2$.

4. **(a)** Substituting $x = 0$ gives $g(0) = 0^2 - 5(0) + 6 = 6$.
(b) Setting $g(x) = 0$ and solving gives $x^2 - 5x + 6 = 0$.
Factoring gives $(x-2)(x-3) = 0$, so $x = 2, 3$.

5. **(a)** Substituting $t = 0$ gives

$$g(0) = \frac{1}{0+2} - 1 = \frac{1}{2} - 1 = -\frac{1}{2}.$$

(b) Setting $g(t) = 0$ and solving gives

$$\begin{aligned} \frac{1}{t+2} - 1 &= 0 \\ \frac{1}{t+2} &= 1 \\ 1 &= t+2 \\ t &= -1. \end{aligned}$$

6. Substituting zero for x gives

$$h(0) = a \cdot 0^2 + b \cdot 0 + c = c.$$

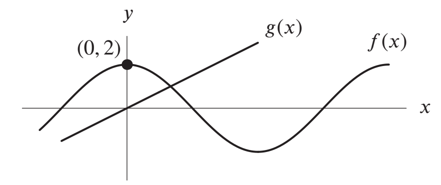
7. To evaluate $p(7)$, we substitute 7 for each r in the formula:

$$p(7) = 7^2 + 5 = 54.$$

8. To evaluate $p(x) + p(8)$, we substitute x and 8 for each r in the formula and add the two expressions:

$$p(x) + p(8) = (x^2 + 5) + (8^2 + 5) = x^2 + 74.$$

9.



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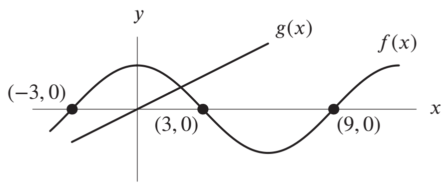

11. Curves cross at $x = 2$. See Figure 2.1. We do not know the y-coordinate of this point.

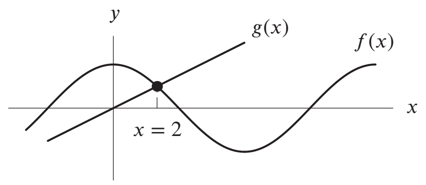

Figure 2.1

12. Graph of $g(x)$ is above graph of $f(x)$ for x to the right of 2. See Figure 2.2.

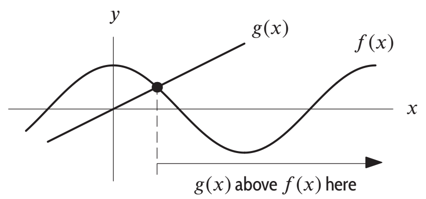

Figure 2.2

13. The solutions to $g(x) = 2$ are all x values that satisfy $2x + 2 = 2$. Subtracting 2 from both sides, we get $2x = 0$, and dividing by 2, we get $x = 0$. Thus there is only one x value, $x = 0$, with $g(x) = 2$.

14. The solutions to $g(x) < 2$ are all x that satisfy $2x + 2 < 2$. Subtracting 2 from both sides, we get $2x < 0$, and then dividing by 2, we get $x < 0$. Therefore, the solutions of $g(x) < 2$ are all x with $x < 0$.

15. First we find where the two graphs intersect. To so this, we solve $x^2 + 2x - 2 = x^2$. Solving, we get

$$\begin{aligned} x^2 + 2x - 2 &= x^2 \\ 2x - 2 &= 0 \\ x &= 1. \end{aligned}$$

We can solve the inequality the same way:

$$\begin{aligned} x^2 + 2x - 2 &< x^2 \\ 2x - 2 &< 0 \\ 2x &< 2 \\ x &< 1. \end{aligned}$$

16. First we find where the two graphs intersect. To find these points, we solve $x^2 + 2x - 2 = 2x + 2$. Solving, we get

$$\begin{aligned} x^2 + 2x - 2 &= 2x + 2 \\ x^2 + 2x &= 2x + 4 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

Using a calculator or computer to graph the curves $y = f(x)$ and $y = g(x)$, as in Figure 2.3, we see that the graph of $f(x)$ lies above the graph of $g(x)$ to the left of $x = -2$ and to the right of $x = 2$. Therefore, the solutions to the inequality $f(x) > g(x)$ are

$$x < -2 \quad \text{or} \quad x > 2.$$

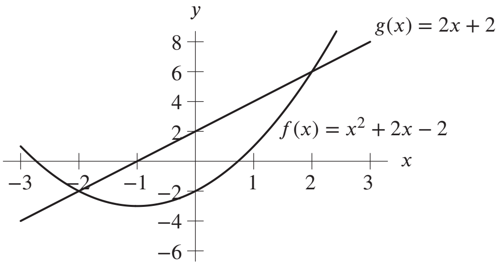

Figure 2.3: The function $f(x) = x^2 + 2x - 2$

Problems

17. The input, t, is the number of months since January 1, and the output, F, is the number of foxes. The expression $g(9)$ represents the number of foxes in the park on October 1. Table 1.3 on page 5 of the text gives $F = 100$ when $t = 9$. Thus, $g(9) = 100$. On October 1, there were 100 foxes in the park.

18. The output $g(t)$ stands for a number of foxes. We want to know in what month there are 75 foxes. Table 1.3 on page 5 of the text tells us that this occurs when $t = 4$ and $t = 8$; that is, in May and in September.

19. **(a)** Table 2.2 in the text shows $f(12) = 158.1$. Since $t = 12$ in the year 2004, we know that a member's salary was \$158,100 in the year 2004.

(b) Solving $f(t) = 169.3$ means finding the year in which a member's salary was \$169,300; it is $t = 16$. Since $t = 16$ in 2008, a member's salary was \$169,300 in 2008.

20. Substituting $\frac{1}{3}$ for x gives

$$f\left(\frac{1}{3}\right) = 3 + 2\left(\frac{1}{3}\right)^2 = 3 + \frac{2}{9} = 3.222.$$

On the other hand,

$$\begin{aligned} f(1) &= 3 + 2(1)^2 = 5 \\ f(3) &= 3 + 2(3)^2 = 21. \end{aligned}$$

So $\dfrac{f(1)}{f(3)} = \dfrac{5}{21} = 0.238$, and we see that

$$f\left(\frac{1}{3}\right) \neq \frac{f(1)}{f(3)}.$$

They are not equal.

21. **(a)** $g(100) = 100\sqrt{100} + 100 \cdot 100 = 100 \cdot 10 + 100 \cdot 100 = 11,000$

(b) $g(4/25) = 4/25 \cdot \sqrt{4/25} + 100 \cdot 4/25 = 4/25 \cdot 2/5 + 16 = 8/125 + 16 = 16.064$

(c) $g(1.21 \cdot 10^4) = g(12100) = (12100)\sqrt{12100} + 100 \cdot (12100) = 2,541,000$

22. **(a)** $g(x) = x^2 + x$
$g(-3x) = (-3x)^2 + (-3x)$
$g(-3x) = 9x^2 - 3x$
(b) $g(1-x) = (1-x)^2 + (1-x) = (1 - 2x + x^2) + (1 - x) = x^2 - 3x + 2$
(c) $g(x+\pi) = (x+\pi)^2 + (x+\pi) = (x^2 + 2\pi x + \pi^2) + (x+\pi) = x^2 + (2\pi + 1)x + \pi^2 + \pi$
(d) $g(\sqrt{x}) = (\sqrt{x})^2 + \sqrt{x} = x + \sqrt{x}$
(e) $g\left(\frac{1}{x+1}\right) = \left(\frac{1}{(x+1)^2}\right) + \frac{1}{x+1} = \frac{1}{(x+1)^2} + \frac{x+1}{(x+1)^2} = \frac{x+2}{(x+1)^2}$
(f) $g(x^2) = (x^2)^2 + x^2 = x^4 + x^2$

23. **(a)** (i) $\frac{\frac{1}{t}}{\frac{1}{t} - 1} = \frac{\frac{1}{t}}{\frac{1-t}{t}} = \frac{1}{t} \cdot \frac{t}{1-t} = \frac{1}{1-t}.$

(ii) $\frac{\frac{1}{t+1}}{\frac{1}{t+1} - 1} = \frac{1}{t+1} \cdot \frac{t+1}{1-t-1} = -\frac{1}{t}.$

(b) Solve $f(x) = \frac{x}{(x-1)} = 3$, so

$$x = 3x - 3$$
$$3 = 2x$$
$$x = \frac{3}{2}.$$

24. **(a)** $f(2) = 2^2 = 4$
(b) $f(2+h) = (2+h)^2 = 4 + 2h + h^2$.
(c) Using the answers to (a) and (b), we have

$$f(2+h) - f(2) = 4 + 2h + h^2 - 4 = 2h + h^2$$

(d) Using the answer to part (c), we have

$$\frac{f(2+h) - f(2)}{h} = \frac{2h + 2h^2}{h}$$

25. Since $g(t+h) = 2(t+h) - 1$ we have

$$\frac{g(t+h) - g(t)}{h} = \frac{2(t+h) - 1 - (2t-1)}{h} = \frac{2t + 2h - 1 - 2t + 1}{h} = \frac{2h}{h}.$$

26. **(a)**

x	-2	-1	0	1	2	3
$h(x)$	0	9	8	3	0	6

(b) From the table, we see that $h(3) = 6$, while $h(1) = 3$. Thus, $h(3) - h(1) = 6 - 3 = 3$.
(c) From the table, we see that $h(2) = 0$, and $h(0) = 8$. Thus, $h(2) - h(0) = 0 - 8 = -8$.
(d) From the table, we see that $h(0) = 8$. Thus, $2h(0) = 2(8) = 16$.
(e) From the table, we see that $h(1) = 3$. Thus, $h(1) + 3 = 3 + 3 = 6$.

27. **(a)** Substituting into $h(t) = -16t^2 + 64t$, we get

$$h(1) = -16(1)^2 + 64(1) = 48$$
$$h(3) = -16(3)^2 + 64(3) = 48$$

Thus the height of the ball is 48 feet after 1 second and after 3 seconds.
(b) The graph of $h(t)$ is in Figure 2.4. The ball is on the ground when $h(t) = 0$. From the graph we see that this occurs at $t = 0$ and $t = 4$. The ball leaves the ground when $t = 0$ and hits the ground at $t = 4$ or after 4 seconds. From the graph we see that the maximum height is 64 ft.

Full file at https://testbankuniv.eu/Functions-Modeling-Change-A-Preparation-for-Calculus-5th-Edition-Connally-Solutions-Manual

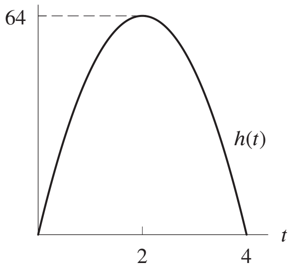

Figure 2.4

28. **(a)** We have $P(0) = 20(0) - 500 = -500$ dollars, which means that if no tickets are sold, the theater will lose 500 dollars.
(b) Setting $P(n) = 0$ and solving, we get

$$\begin{aligned} 20n - 500 &= 0 \\ 20n &= 500 \\ n &= 25. \end{aligned}$$

The theater will make a profit of 0 if 25 tickets are sold.
(c) The quantity $P(100)$ represents the profit, in dollars, made by the theater if 100 tickets are sold.

29. **(a)** Substituting $t = 0$ gives $v(0) = 0^2 - 2(0) = 0 - 0 = 0$.
(b) To find when the object has velocity equal to zero, we solve the equation

$$\begin{aligned} t^2 - 2t &= 0 \\ t(t-2) &= 0 \\ t &= 0 \quad \text{or} \quad t = 2. \end{aligned}$$

Thus the object has velocity zero at $t = 0$ and at $t = 2$.
(c) The quantity $v(3)$ represents the velocity of the object at time $t = 3$. Its units are ft/sec.

30. **(a)** The car's position after 2 hours is denoted by the expression $s(2)$. The position after 2 hours is

$$s(2) = 11(2)^2 + 2 + 100 = 44 + 2 + 100 = 146.$$

(b) This is the same as asking the following question: "For what t is $v(t) = 65$?"
(c) To find out when the car is going 67 mph, we set $v(t) = 67$. We have

$$\begin{aligned} 22t + 1 &= 67 \\ 22t &= 66 \\ t &= 3. \end{aligned}$$

The car is going 67 mph at $t = 3$, that is, 3 hours after starting. Thus, when $t = 3$, $S(3) = 11(3^2) + 3 + 100 = 202$, so the car's position when it is going 67 mph is 202 miles.

31. **(a)** From the graph, we estimate that

$$f(t+5) = f(6) = 18.$$

The quantity $f(t+5)$ represents the speed of the car, in feet per second, 5 seconds later than time t.
(b) From the graph, we estimate that

$$f(t) + 5 = f(1) + 5 = 96.$$

The quantity $f(t) + 5$ represents a speed, in feet per second, that is 5 feet per second faster than the speed of the car at time t.
(c) From the graph, we estimate that $f(4.3) = 40$, so if $f(t+2) = 40$, we must have $t + 2 = 4.3$, or $t = 2.3$ seconds. This represents the time 2 seconds prior to when the car has slowed to 40 feet per second.
(d) Note that the equation $f(t) + 10 = 40$ is equivalent to the equation $f(t) = 30$, and from the graph, we estimate that $f(5) = 30$; therefore, we must have $t = 5$ seconds. This represents the time at which the car has slowed to 30 feet per second.

32. **(a)** From Figure 2.5, we see that $P = (b, a)$ and $Q = (d, e)$.

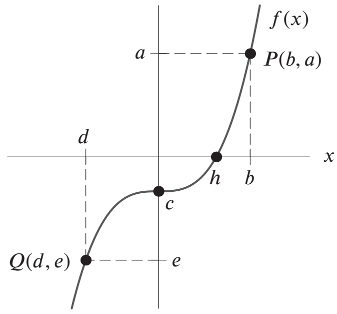

Figure 2.5

(b) To evaluate $f(b)$, we want to find the y-value when the x-value is b. Since (b, a) lies on this graph, we know that the y-value is a, so $f(b) = a$.

(c) To solve $f(x) = e$, we want to find the x-value for a y-value of e. Since (d, e) lies on this curve, $x = d$ is our solution.

(d) To solve $z = f(x)$, we need first to find a value for z; in other words, we need first to solve for $f(z) = c$. Since $(0, c)$ lies on this graph, we know that $z = 0$. Now we need to solve $0 = f(x)$ by finding the point whose y-value is 0. That point is $(h, 0)$, so $x = h$ is our solution.

(e) We know that $f(b) = a$ and $f(d) = e$. Thus, if $f(b) = -f(d)$, we know that $a = -e$.

33. **(a)** Her tax is \$4650.63 on the first \$77,150 plus 6.65% of income over \$77,150, which is \$88,000 − 77,150 = \$10,850. Thus:

$$\text{Tax owed } = \$4650.63 + 0.0665(\$10{,}850) = \$4650.63 + \$721.53 = \$5372.16.$$

(b) Her taxable income, $T(x)$, is 80% of her total income, or 80% of x. So $T(x) = 0.8x$.

(c) Her tax owed is \$4650.63 plus 6.65% of her taxable income over \$77,150. Since her taxable income is $0.8x$, her taxable income over \$77,850 is $0.8x - 77{,}150$. Therefore,

$$L(x) = 4650.63 + 0.0665(0.8x - 77{,}150),$$

so multiplying out and simplifying, we obtain

$$L(x) = 0.0532x - 479.84$$

(d) Evaluating for $x = \$110{,}000$, we have

$$\begin{aligned} L(110{,}000) &= 0.0532(110{,}000) - 479.84 \\ &= 5372.16. \end{aligned}$$

The values are the same.

34. **(a)** (i) From the table, $N(150) = 6$. When 150 students enroll, there are 6 sections.

(ii) Since $N(75) = 4$ and $N(100) = 5$, and 80 is between 75 and 100 students, we choose the higher value for $N(s)$. So $N(80) = 5$. When 80 students enroll, there are 5 sections.

(iii) The quantity $N(55.5)$ is not defined, since 55.5 is not a possible number of students.

(b) (i) The table gives $N(s) = 4$ sections for $s = 75$ and $s = 50$. For any integer between those in the table, the section number is the higher value. Therefore, for $50 \le s \le 75$, we have $N(s) = 4$ sections. We do not know what happens if $s < 50$.

(ii) First evaluate $N(125) = 5$. So we solve the equation $N(s) = 5$ for s. There are 5 sections when enrollment is between 76 and 125 students.

35. **(a)** We calculate the values of $f(x)$ and $g(x)$ using the formulas given in Table 2.1.

Table 2.1

x	−2	−1	0	1	2
$f(x)$	6	2	0	0	2
$g(x)$	6	2	0	0	2

The pattern is that $f(x) = g(x)$ for $x = -2, -1, 0, 1, 2$. Based on this, we might speculate that f and g are really the same function. This is, in fact, the case, as can be verified algebraically:

$$\begin{aligned} f(x) &= 2x(x-3) - x(x-5) \\ &= 2x^2 - 6x - x^2 + 5x \\ &= x^2 - x \\ &= g(x). \end{aligned}$$

Their graphs are the same, and are shown in Figure 2.6.

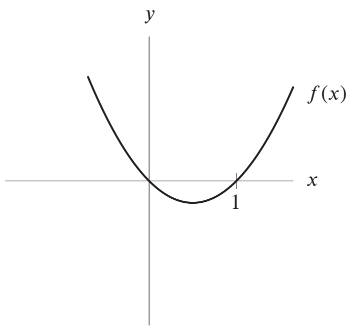

Figure 2.6

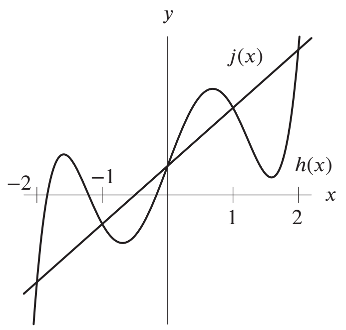

Figure 2.7

(b) Using the formulas for $h(x)$ and $j(x)$, we obtain Table 2.2.

Table 2.2

x	-2	-1	0	1	2
$h(x)$	-3	-1	1	3	5
$j(x)$	-3	-1	1	3	5

The pattern is that $h(x) = j(x)$ for $x = -2, -1, 0, 1, 2$. Based on this, we might speculate that h and j are really the same function. The graphs of these functions are shown in Figure 2.7. We see that the graphs share only the points of the table and are thus two different functions.

36. (a) To evaluate $f(1)$, we need to find the value of f which corresponds to $x = 1$. Looking in the table, we see that that value is 2. So we can say $f(1) = 2$. Similarly, to find $g(3)$, we see in the table that the value of g which corresponds to $x = 3$ is 4. Thus, we know that $g(3) = 4$.

(b) The values of $f(x)$ increase by 3 as x increases by 1. For $x > 1$, the values of $g(x)$ are consecutive perfect squares. The entries for $g(x)$ are symmetric about $x = 1$. In other words, when $x < 1$ the values of $g(x)$ are the same as the values when $x > 1$, but the order is reversed.

(c) Since the values of $f(x)$ increase by 3 as x increases by 1 and $f(4) = 11$, we know that $f(5) = 11+3 = 14$. Similarly, $f(x)$ decreases by three as x goes down by one. Since $f(-1) = -4$, we conclude that $f(-2) = -4 - 3 = -7$.

The values of $g(x)$ are consecutive perfect squares. Since $g(4) = 9$, then $g(5)$ must be the next perfect square which is 16, so $g(5) = 16$. Since the values of $g(x)$ are symmetric about $x = 1$, the value of $g(-2)$ will equal $g(5)$ (since -2 and 4 are both a distance of 3 units from 1). Thus, $g(-2) = g(4) = 9$.

(d) To find a formula for $f(x)$, we begin by observing that $f(0) = -1$, so the value of $f(x)$ that corresponds to $x = 0$ is -1. We know that the value of $f(x)$ increases by 3 as x increases by 1, so

$$\begin{aligned} f(1) &= f(0) + 3 = -1 + 3 \\ f(2) &= f(1) + 3 = (-1 + 3) + 3 = -1 + 2 \cdot 3 \\ f(3) &= f(2) + 3 = (-1 + 2 \cdot 3) + 3 = -1 + 3 \cdot 3 \\ f(4) &= f(3) + 3 = (-1 + 3 \cdot 3) + 3 = -1 + 4 \cdot 3. \end{aligned}$$

The pattern is

$$f(x) = -1 + x \cdot 3 = -1 + 3x.$$

We can check this formula by choosing a value for x, such as $x = 4$, and use the formula to evaluate $f(4)$. We find that $f(4) = -1 + 3(4) = 11$, the same value we see in the table.

Since the values of $g(x)$ are all perfect squares, we expect the formula for $g(x)$ to have a square in it. We see that x^2 is not quite right since the table for such a function would look like Table 2.3.

Table 2.3

x	-1	0	1	2	3	4
x^2	1	0	1	4	9	16

But this table is very similar to the one that defines g. In order to make Table 2.3 look identical to the one given in the problem, we need to subtract 1 from each value of x so that $g(x) = (x-1)^2$. We can check our formula by choosing a value for x, such as $x = 2$. Using our formula to evaluate $g(2)$, we have $g(2) = (2-1)^2 = 1^2 = 1$. This result agrees with the value given in the problem.

37. $r(0.5s_0)$ is the wind speed at a half the height above ground of maximum wind speed.

38. In $r(s) = 0.75v_0$, the variable s is the height (or heights) at which the wind speed is 75% of the maximum wind speed.

39. **(a)** $h(1) = (1)^2 + b(1)^2 + c = b + c + 1$

(b) Substituting $b+1$ for x in the formula for $h(x)$:

$$\begin{aligned} h(b+1) &= (b+1)^2 + b(b+1) + c \\ &= (b^2 + 2b + 1) + b^2 + b + c \\ &= 2b^2 + 3b + c + 1 \end{aligned}$$

40. $f(a) = \dfrac{a \cdot a}{a+a} = \dfrac{a^2}{2a} = \dfrac{a}{2}.$

41. $f(1-a) = \dfrac{a(1-a)}{a+(1-a)} = a(1-a) = a - a^2$

42. $f\left(\dfrac{1}{1-a}\right) = \dfrac{a\dfrac{1}{1-a}}{a + \dfrac{1}{1-a}} = \dfrac{\dfrac{a}{1-a}}{\dfrac{a(1-a)+1}{1-a}} = \dfrac{a}{1-a} \cdot \dfrac{1-a}{a-a^2+1} = \dfrac{a}{a-a^2+1}.$

43. **(a)** See Figure 2.8.

(b) See Figure 2.8.

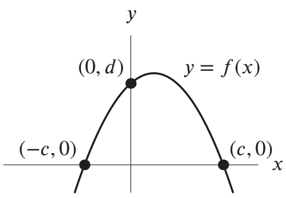

Figure 2.8

44. **(a)** To evaluate $f(2)$, we determine which value of I corresponds to $w = 2$. Looking at the graph, we see that $I \approx 7$ when $w = 2$. This means that ≈ 7000 people were infected two weeks after the epidemic began.

(b) The height of the epidemic occurred when the largest number of people were infected. To find this, we look on the graph to find the largest value of I, which seems to be approximately 8.5, or 8500 people. This seems to have occurred when $w = 4$, or four weeks after the epidemic began. We can say that at the height of the epidemic, at $w = 4$, $f(4) = 8.5$.

(c) To solve $f(x) = 4.5$, we must find the value of w for which $I = 4.5$, or 4500 people were infected. We see from the graph that there are actually two values of w at which $I = 4.5$, namely $w \approx 1$ and $w \approx 10$. This means that 4500 people were infected after the first week when the epidemic was on the rise, and that after the tenth week, when the epidemic was slowing, 4500 people remained infected.

(d) We are looking for all the values of w for which $f(w) \geq 6$. Looking at the graph, this seems to happen for all values of $w \geq 1.5$ and $w \leq 8$. This means that more than 6000 people were infected starting in the middle of the second week and lasting until the end of the eighth week, after which time the number of infected people fell below 6000.

45. This represents the change in average hurricane intensity at average Caribbean Sea surface temperature after CO_2 levels rise to future projected levels.

46. This represents the change in average hurricane intensity at current CO_2 levels if sea surface temperature rises by 1°C.

Solutions for Section 2.2

Skill Refresher

S1. The function is undefined when the denominator is zero. Therefore, $x - 3 = 0$ tells us the function is undefined for $x = 3$.

S2. The function is undefined when the denominator is zero. Therefore, $x - 3 = 0$ tells us the function is undefined for $x = 3$ But it is also undefined at $x = 0$.

S3. Because of the square root the function is undefined when $x - 15$ is negative. That is, when $x < 15$.

S4. Because of the square root the function is undefined when $15 - x$ is negative. That is, when $x > 15$.

S5. Adding 8 to both sides of the inequality we get $x > 8$.

S6. First we subtract 5 from both sides to get $-x > -5$. Then multiplying both sides by -1 and changing the direction of the inequality, we get $x < 5$.

S7. Divide by -3 with a direction change, and then add 4.

$$\begin{aligned} -3(n-4) &> 12 \\ \frac{-3(n-4)}{-3} &< \frac{12}{-3} \\ n-4 &< -4 \\ n-4+4 &< -4+4 \\ n &< 0. \end{aligned}$$

S8. Subtract 24, then divide by -4 with a direction change:

$$\begin{aligned} 12 &\leq 24-4a \\ 12-24 &\leq 24-4a-24 \\ -12 &\leq -4a \\ \frac{-12}{-4} &\geq \frac{-4a}{-4} \\ 3 &\geq a. \end{aligned}$$

Or, equivalently, $a \leq 3$.

S9. $x^2 - 25 > 0$ is true when $x > 5$ or $x < -5$.

S10. We have $36 - x^2 \geq 0$ when $-6 \leq x \leq 6$

S11. Adding $2a^2$ to both sides gives $12 \leq 3a^2$. Then dividing by 3 gives $4 \leq a^2$. This is true when $a \geq 2$ and $a \leq -2$.

S12. Subtracting y^2 from both sides and adding 3 gives $2y^2 > 18$. Then dividing by 2 gives $y^2 > 9$. This is true when $y > 3$ or $y < -3$.

Exercises

1. The domain is $1 \le x \le 7$. The range is $2 \le f(x) \le 18$.

2. The domain is $2 \le x \le 6$. The range is $1 \le f(x) \le 3$.

3. The domain is $1 \le x \le 5$. The range is $1 \le f(x) \le 6$.

4. The domain is $0 \le x \le 4$. The range is $0 \le f(x) \le 2$.

5. The graph of $f(x) = 1/x$ for $-2 \le x \le 2$ is shown in Figure 2.9. From the graph, we see that $f(x) = -(1/2)$ at $x = -2$. As we approach zero from the left, $f(x)$ gets more and more negative. On the other side of the y-axis, $f(x) = (1/2)$ at $x = 2$. As x approaches zero from the right, $f(x)$ grows larger and larger. Thus, on the domain $-2 \le x \le 2$, the range is $f(x) \le -(1/2)$ or $f(x) \ge (1/2)$.

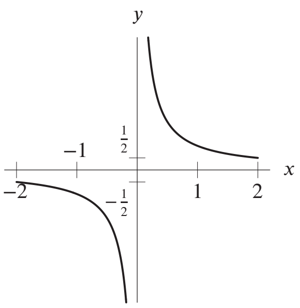

Figure 2.9

6. The graph of $f(x) = 1/x^2$ for $-1 \le x \le 1$ is shown in Figure 2.10. From the graph, we see that $f(x) = 1$ at $x = -1$ and $x = 1$. As we approach 0 from 1 or from -1, the graph increases without bound. The lower limit of the range is 1, while there is no upper limit. Thus, on the domain $-1 \le x \le 1$, the range is $f(x) \ge 1$.

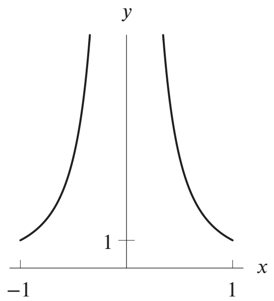

Figure 2.10

7. The graph of $f(x) = x^2 - 4$ for $-2 \le x \le 3$ is shown in Figure 2.11. From the graph, we see that $f(x) = 0$ at $x = -2$, that $f(x)$ decreases down to -4 at $x = 0$, and then increases to $f(x) = 3^2 - 4 = 5$ at $x = 3$. The minimum value of $f(x)$ is -4, while the maximum value is 5. Thus, on the domain $-2 \le x \le 3$, the range is $-4 \le f(x) \le 5$.

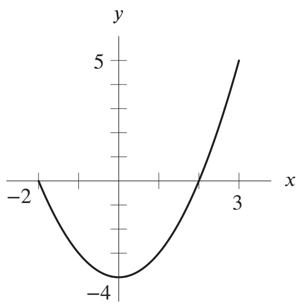

Figure 2.11

8. The graph of $f(x) = \sqrt{9 - x^2}$ for $-3 \le x \le 1$ is shown in Figure 2.12. From the graph, we see that $f(x) = 0$ at $x = -3$, and that $f(x)$ increases to a maximum value of 3 at $x = 0$, and then decreases to a value of $f(x) = \sqrt{9 - 1^2} \approx 2.83$ or $= 2\sqrt{2}$ at $x = 1$. Thus, on the domain $-3 \le x \le 1$, the range is $0 \le f(x) \le 3$.

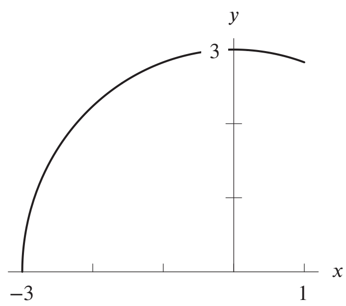

Figure 2.12

9. The domain is all real numbers except those which do not yield an output value. The expression $1/(x+3)$ is defined for any real number x except -3, since for $x = -3$ the denominator of $f(x)$, $x + 3$, is 0 and division by 0 is undefined. Therefore, the domain of $f(x)$ is all real numbers $\neq -3$.

10. Since division by 0 is undefined, $p(t)$ is defined for any real number except for t, for which the denominator is equal to 0. Solving the equation $t^2 - 4 = 0$, we get $t = \pm 2$. So the domain is all real numbers $\neq \pm 2$.

11. $f(t)$ is not defined when $3t + 9 = 0$. So the domain is all real numbers $\neq -3$.

12. Since $q^4 + 2$ cannot be equal to zero for any real number, the domain is all real numbers.

13. To evaluate $f(x)$, we must have $9 + x > 0$. Thus

$$\text{Domain: } x > -9.$$

14. $y(t)$ is not defined when $t^4 = 0$. So the domain is all real numbers $\neq 0$.

15. For $f(x)$ to be defined, the expression $x^2 - 4$, found inside the square root sign, must always be nonnegative. This happens when $x \ge 2$ or $x \le -2$. So the domain is all real numbers x such that $x \ge 2$ or $x \le -2$.

16. We can take the cube root of any number, so the domain is all real numbers.

17. Any number can be squared, so the domain is all real numbers.

18. To evaluate $t(a)$, we must have $a - 2 \ge 0$. So the domain is all real numbers ≥ 2.

19. Since $m(q)$ is a linear function, the domain of $m(q)$ is all real numbers. For any value of $m(q)$ there is a corresponding value of q. So the range is also all real numbers.

20. The square root function is only defined for non-negative values, that is when $15 - 4x \ge 0$, thus we require that $x \le 15/4$. The domain is $x \le 15/4$. The values of the square root function are all non-negative, so the range is $f(x) \ge 0$.

21. To evaluate $f(x)$, we must have $x - 4 > 0$. Thus

$$\text{Domain: } x > 4.$$

To find the range, we want to know all possible output values. We solve the equation $y = f(x)$ for x in terms of y. Since

$$y = \frac{1}{\sqrt{x-4}},$$

squaring gives

$$y^2 = \frac{1}{x-4},$$

and multiplying by $x - 4$ gives

$$\begin{aligned} y^2(x-4) &= 1 \\ y^2x - 4y^2 &= 1 \\ y^2x &= 1 + 4y^2 \\ x &= \frac{1+4y^2}{y^2}. \end{aligned}$$

This formula tells us how to find the x-value which corresponds to a given y-value. The formula works for any y except $y = 0$ (which puts a 0 in the denominator). We know that y must be positive, since $\sqrt{x-4}$ is positive, so we have

$$\text{Range: } y > 0.$$

22. To evaluate $f(x)$, we must have $9 - x > 0$. Thus

$$\text{Domain: } x < 9.$$

To find the range, we want to know all possible output values. We solve the equation $y = f(x)$ for x in terms of y. Since

$$y = \frac{1}{\sqrt{9-x}},$$

squaring gives

$$y^2 = \frac{1}{9-x},$$

and multiplying by $9 - x$ gives

$$\begin{aligned} y^2(9-x) &= 1 \\ 9y^2 - y^2x &= 1 \\ y^2x &= 9y^2 - 1 \\ x &= \frac{9y^2-1}{y^2}. \end{aligned}$$

This formula tells us how to find the x-value which corresponds to a given y-value. The formula works for any y except $y = 0$ (which puts a 0 in the denominator). We know that y must be positive, since $\sqrt{9-x}$ is positive, so we have

$$\text{Range: } y > 0.$$

Problems

23. The function $f(x)$ is not defined when $x = a$. So $a = 3$.

24. The function $p(t)$ is not defined when $2t - a = 0$ and $t + b = 0$. So $p(t)$ is not defined when $t = a/2$ and $t = -b$. A possible answer is $a/2 = 4$, namely $a = 8$ and $b = -5$.

25. The function $n(q)$ can be evaluated only when $q^2 + a \geq 0$. Since, when q is 0, any $a < 0$ gives $q^2 + a < 0$, we know that a must be greater than or equal to zero. Furthermore, for all $a \geq 0$, we have $q^2 + a \geq 0$. Hence the domain of $n(q)$ is all real numbers for any $a \geq 0$. So any nonnegative real number, such as 0, $\frac{1}{2}$, $\sqrt{2}$, or 77 is a possible value of a giving $n(q)$ a domain of all real numbers.

26. The function $m(r)$ is defined when $r - a \geq 0$. So the domain of $m(r)$ is all real numbers $\geq a$. Hence $a = -3$.

27. One way to do this is to combine two operations, one of which forces x to be less than -4, the other of which forces x not to equal -2. For example:

A function such as $y = \sqrt{x+4}$ is undefined for $x < -4$, because the input of the square root operation is negative for these x-values.

A function such as $y = 1/(x+2)$ is undefined for $x = -2$.

Combining two functions such as these, for example by adding or multiplying them, yields a function with the required domain. Thus, possible functions include

$$y = \frac{1}{x+2} + \sqrt{x+4} \qquad \text{or} \qquad y = \frac{\sqrt{x+4}}{x+2}.$$

Other answers are possible.

28. One way to do this is to combine two operations, one of which forces x to be negative, the other of which forces x not to equal -5. For example:

A function such as $y = 1/(x+5)$ forces $x \neq 5$ so that the denominator is not 0.

A function such as $y = 1/\sqrt{-x}$ forces x to be negative so that the input to the square root is not negative, and the denominator is not 0.

We combine these functions, for example as

$$y = \frac{1}{x+5} + \frac{1}{\sqrt{-x}} \qquad \text{or} \qquad y = \frac{1}{(x+5)\sqrt{-x}}.$$

Other answers are possible.

29. Since the restaurant opens at 2 pm, $t = 0$, and closes at 2 am, $t = 12$, a reasonable domain is $0 \le t \le 12$.

Since there cannot be fewer than 0 clients in the restaurant and 200 can fit inside, the range is $0 \le f(t) \le 200$.

30. A possible graph of gas mileage (in miles per gallon, mpg) is shown in Figure 2.13. The function shown has a domain $0 \le x \le 120$ mph, as the car cannot have a negative speed and is not likely to go faster than 120 mph. The range of the function shown is $0 \le y \le 40$ mpg. A wide variety of other answers is possible.

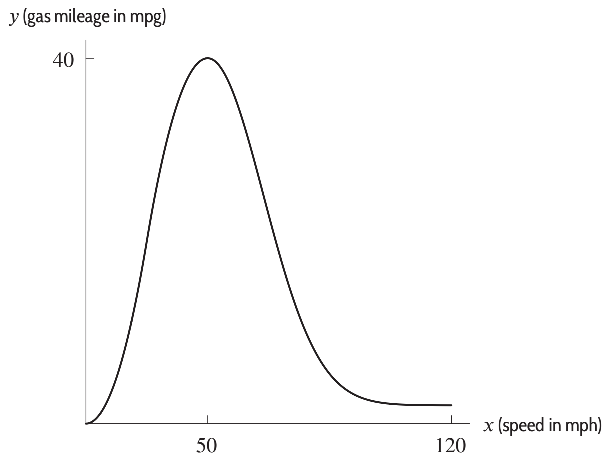

Figure 2.13

31. We know that the theater can hold anywhere from 0 to 200 people. Therefore the domain of the function is the integers, n, such that $0 \le n \le 200$.

We know that each person who enters the theater must pay \$4.00. Therefore, the theater makes $(0) \cdot (\$4.00) = 0$ dollars if there is no one in the theater, and $(200) \cdot (\$4.00) = \$800.00$ if the theater is completely filled. Thus the range of the function would be the integer multiples of 4 from 0 to 800. (That is, $0, 4, 8, \ldots$.)

The graph of this function is shown in Figure 2.14.

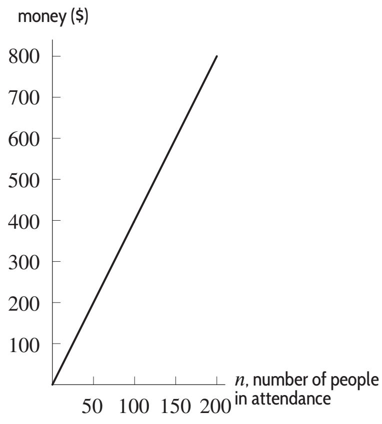

Figure 2.14

32. **(a)** (i) From the table we find that a 200 lb person uses 5.4 calories per minute while walking. So a half-hour, or 30-minute, walk burns $30(5.4) = 162$ calories.

(ii) The number of calories used per minute increases with the person's weight. Weight is the independent variable and number of calories burned is the dependent variable.

(b) (i) Since the function is approximately linear, its equation is $c = b + mw$, where c is the number of calories and w is weight. Using the first two values in the table, the slope is

$$m = \frac{3.2 - 2.7}{120 - 100} = \frac{0.5}{20} = 0.025 \text{ cal/lb.}$$

Using the point $(100, 2.7)$, we have

$$2.7 = b + 0.025(100)$$
$$b = 0.2.$$

So the equation is $c = 0.2 + 0.025w$. See Figure 2.15. All the values given lie on this line with the exception of the last two, which are slightly above it.

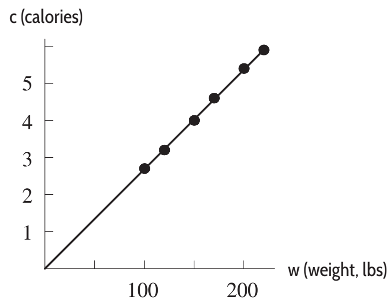

Figure 2.15

(ii) The domain and range values must be positive: Domain $0 < w$; Range $0 < c$.

The table gives weights between 100 to 220 lb and calories per minute between 2.7 and 5.9. We do not know whether the trend in the table continues outside these values, so reasonable restrictions are

$$\text{Domain is } 100 \leq w \leq 220$$
$$\text{Range is } 2.7 \leq c \leq 5.9.$$

Other answers are possible.

(iii) Evaluating the function at $w = 135$,

$$\text{Calories per minute } = 0.2 + 0.025(135) \approx 3.6.$$

33. We can put in any number for x except zero, which makes $1/x$ undefined. We note that as x approaches infinity or negative infinity, $1/x$ approaches zero, though it never arrives there, and that as x approaches zero, $1/x$ goes to negative or positive infinity. Thus, the range is all real numbers except a.

34. Since $(x-b)^{1/2} = \sqrt{x-b}$, we know that $x - b \geq 0$. Thus, $x \geq b$. If $x = b$, then $(x-b)^{1/2} = 0$, which is the minimum value of $\sqrt{x-b}$, since it can't be negative. Thus, the range is all real numbers greater than or equal to 6.

35. **(a)** We see that the 6^{th} listing has a last digit of 9. Thus, $f(6) = 9$.
(b) The domain of the telephone directory function is $n = 1, 2, 3, \ldots, N$, where N is the total number of listings in the directory. We could find the value of N by counting the number of listings in the phone book.
(c) The range of this function is $d = 0, 1, 2, \ldots, 9$, because the last digit of any listing must be one of these numbers.

36. **(a)** $r(0) = 800 - 40(0) = 800$ means water is entering the reservoir at 800 gallons per second at time $t = 0$. Since we don't know how much water was in the reservoir originally, this is not the amount of water in the reservoir.
$r(15) = 800 - 40(15) = 800 - 600 = 200$ means water is entering the reservoir at 200 gallons per second at time $t = 15$.
$r(25) = 800 - 40(25) = 800 - 1000 = -200$ means water is leaving the reservoir at 200 gallons per second at time $t = 25$.
(b) The intercepts occur at $(0, 800)$ and $(20, 0)$. The first tells us that the water is initially flowing in at the rate of 800 gallons per second. The other tells us that at 20 seconds, the flow has stopped.
The slope is $(800-0)/(0-20) = 800/-20 = -40$. This means that the rate at which water enters the reservoir decreases by 40 gallons per second each second. The water is flowing in at a decreasing rate.

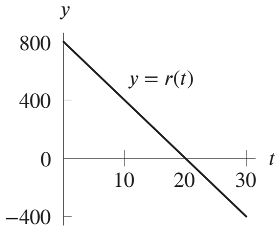

Figure 2.16

(c) The reservoir has more and more water when the rate is positive, because then water is being added. Water is being added until $t = 20$ when it starts flowing out. This means at $t = 20$, the most water is in the reservoir. The reservoir has water draining out between $t = 20$ and $t = 30$, but this amount is not as much as the water that entered between $t = 0$ and $t = 20$. Thus, the reservoir had the least amount of water at the beginning when $t = 0$. Remember the graph shows the rate of flow, not the amount of water in the reservoir.
(d) The domain is the number of seconds specified; $0 \leq t \leq 30$. The rate varies from 800 gallons per second at $t = 0$ to -400 gallons per second at $t = 30$, so the range is $-400 \leq r(t) \leq 800$.

37. **(a)** Substituting $t = 0$ into the formula for $p(t)$ shows that $p(0) = 50$, meaning that there were 50 rabbits initially. Using a calculator, we see that $p(10) \approx 131$, which tells us there were about 131 rabbits after 10 months. Similarly, $p(50) \approx 911$ means there were about 911 rabbits after 50 months.
(b) The graph in Figure 2.17 tells us that the rabbit population grew quickly at first but then leveled off at about 1000 rabbits after around 75 months or so. It appears that the rabbit population increased until it reached the island's capacity.

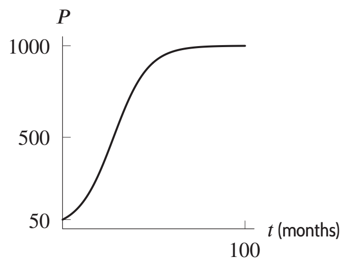

Figure 2.17

(c) From the graph in Figure 2.17, we see that the range is $50 \le p(t) \le 1000$. This tells us that (for $t \ge 0$) the number of rabbits is no less than 50 and no more than 1000.

(d) The smallest population occurred when $t = 0$. At that time, there were 50 rabbits. As t gets larger and larger, $(0.9)^t$ gets closer and closer to 0. Thus, as t increases, the denominator of

$$p(t) = \frac{1000}{1 + 19(0.9)^t}$$

decreases. As t increases, the denominator $1 + 19(0.9)^t$ gets close to 1 (try $t = 100$, for example). As the denominator gets closer to 1, the fraction gets closer to 1000. Thus, as t gets larger and larger, the population gets closer and closer to 1000. Thus, the range is $50 \le p(t) < 1000$.

38. (a) We can add as much copper to our alloy as we like, so, since positive x-values represent quantities of added copper, x can be as big as we please. But, since the alloy starts off with only 3 kg of copper, we can remove no more than this. Therefore, the domain of f is $x \ge -3$.

For the range of f, note that the output of f is a percentage of copper. Since the alloy can contain no less than 0% copper (as would be the case if all 3 kg were removed), we see that $f(x)$ must be greater than (or equal to) 0%. On the other hand, no matter how much copper we add, the alloy will always contain 6 kg of tin. Thus, we can never obtain a pure, 100%-copper alloy. This means that if $y = f(x)$,

$$0\% \le y < 100\%,$$

or

$$0 \le y < 1.$$

(b) By definition, $f(x)$ is the percentage of copper in the bronze alloy after x kg of copper are added (or removed). We have

$$\begin{matrix}\text{Percentage of copper} \\ \text{in the bronze alloy}\end{matrix} = \frac{\text{quantity of copper in the alloy}}{\text{total quantity of alloy}}.$$

Since x is the quantity of copper added or removed, this gives

$$f(x) = \frac{\text{initial quantity of copper} + x}{\text{initial quantity of alloy} + x},$$

and since the original 9 kg of alloy contained 3 kg of copper, we have

$$f(x) = \frac{3 + x}{9 + x}.$$

(c) If we think of the formula $f(x) = (3 + x)/(9 + x)$ as defining a function, but not as a model of an alloy of bronze, then the way we think about its domain and range changes. For example, we no longer need to ask, "Does this x-value make sense in the context of the model?" We need only ask "Is $f(x)$ algebraically defined for this value of x?" or "If we use this x-value for input, will there be a corresponding y-value as output?"

For the domain of f, we see that $y = (3 + x)/(9 + x)$ is defined for any x-value other than $x = -9$. Thus, the domain of f is any value of x such that $x \ne -9$.

To find the range of this function, we solve $y = f(x)$ for x in terms of y:

$$\begin{aligned}
y &= \frac{3 + x}{9 + x} \\
y(9 + x) &= 3 + x && \text{(multiply both sides by denominator)} \\
9y + xy &= 3 + x && \text{(expand parentheses)} \\
xy - x &= 3 - 9y && \text{(collect all terms with } x \text{ at left)} \\
x(y - 1) &= 3 - 9y && \text{(factor out } x) \\
x &= \frac{3 - 9y}{y - 1} && \text{(divide by } y - 1).
\end{aligned}$$

In solving for x, at the last step we had to divide by $y - 1$. This is valid if and only if $y \ne 1$, for otherwise we would be dividing by zero. There is no x-value resulting in a y-value of 1. Consequently, the range of f is any value of y such that $y \ne 1$.

Notice the difference between this situation and the situation where f is being used as a model for bronze.

39. **(a)** If $V = \pi r^2 h$ and $V = 355$, then $\pi r^2 h = 355$. So $h = (355)/(\pi r^2)$. Thus, since

$$A = 2\pi r^2 + 2\pi r h,$$

we have

$$A = 2\pi r^2 + 2\pi r\left(\frac{355}{\pi r^2}\right),$$

and

$$A = 2\pi r^2 + \frac{710}{r}.$$

(b)

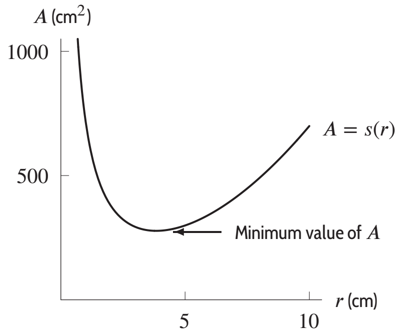

Figure 2.18: Graph of $A(r)$ for $0 < r \leq 10$

(c) The domain is any positive value or $r > 0$, because (in practice) a cola can could have as large a radius as you wanted (it would just have to be very short to maintain its 12 oz size). From the graph in (b), the value of A is never less than about 277.5 cm^2. Thus, the range is $A > 277.5$ cm^2 (approximately).

(d) They need a little more than 277.5 cm^2 per can. The minimum A-value occurs (from graph) at $r \approx 3.83$ cm, and since $h = 355/\pi r^2$, $h \approx 7.7$ cm.

(e) Since the radius of a real cola can is less than the value required for the minimum value of A, it must use more aluminum than necessary. This is because the minimum value of A has $r \approx 3.83$ cm and $h \approx 7.7$ cm. Such a can has a diameter of $2r$ or 7.66 cm. This is roughly equal to its height—holding such a can would be difficult. Thus, real cans are made with slightly different dimensions.

Solutions for Section 2.3

Skill Refresher

S1. Since the point zero is not included, this graph represents $x > 0$.

S2. Since the point -3 is included, the graph represents $x \leq -3$.

S3. Since both end points of the interval are solid dots, this graph represents $x \geq 2$ and $x \leq 3$. Combining these two inequalities, we have $2 \leq x \leq 3$.

S4. Since the point zero is not included and the point 4 is included, this graph represents $x > 0$ and $x \leq 4$. Combining these two, we have $0 < x \leq 4$.

S5. Since both end points of the interval are solid dots, this graph represents $x \leq -1$ or $x \geq 2$.

S6. Since the point -1 is not included and the point 4 is included, this graph represents $x < -1$ or $x \geq 4$.

S7. Domain: $2 \leq x < 6$ and Range: $3 \leq x < 5$.

S8. Domain: $1 < x < 7$ and Range: $-8 < x < -1$.

S9. Domain: $-2 \leq x \leq 3$ and range: $-2 \leq x \leq 3$.

S10. Domain: $-2 \leq x < 4$ and Range: $1 \leq x \leq 5$.

Exercises

1. $f(x) = \begin{cases} -1, & -1 \le x < 0 \\ 0, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \end{cases}$ is shown in Figure 2.19.

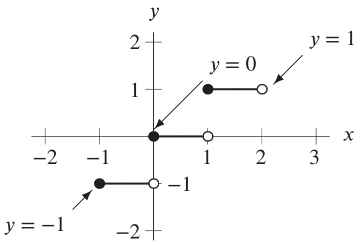

Figure 2.19

2. $f(x) = \begin{cases} x+1, & -2 \le x < 0 \\ x-1, & 0 \le x < 2 \\ x-3, & 2 \le x < 4 \end{cases}$ is shown in Figure 2.20.

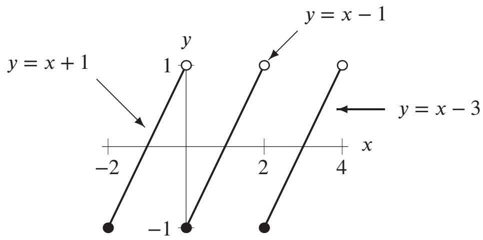

Figure 2.20

3. The graph of $f(x) = \begin{cases} x+4, & x \le -2 \\ 2, & -2 < x < 2 \\ 4-x, & x \ge 2 \end{cases}$ is shown in Figure 2.21.

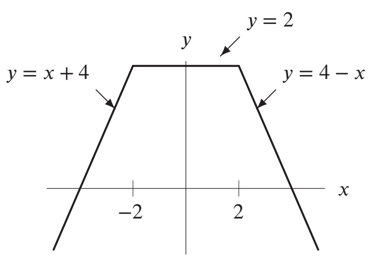

Figure 2.21

4. The graph of $f(x) = \begin{cases} x^2, & x \le 0 \\ \sqrt{x}, & 0 < x < 4 \\ x/2, & x \ge 4 \end{cases}$ is shown in Figure 2.22.

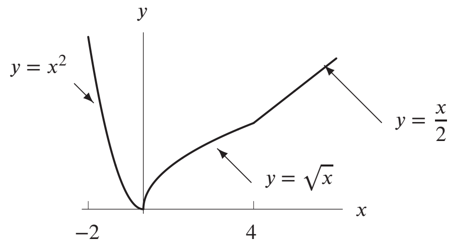

Figure 2.22

5. We find the formulas for each of the lines. For the first, we use the two points we have, $(1, 4)$ and $(3, 2)$. We find the slope: $(2 - 4)/(3 - 1) = -1$. Using the slope of -1, we solve for the y-intercept:

$$\begin{aligned} 4 &= b - 1 \cdot 1 \\ 5 &= b. \end{aligned}$$

Thus, the first line is $y = 5 - x$, and it is for the part of the function where $x < 3$. Notice that we do not use this formula for the value $x = 3$.

We follow the same method for the second line, using the points $(3, \frac{1}{2})$ and $(5, \frac{3}{2})$. We find the slope: $(\frac{3}{2} - \frac{1}{2})/(5-3) = \frac{1}{2}$. Using the slope of $\frac{1}{2}$, we solve for the y-intercept:

$$\begin{aligned} \frac{1}{2} &= b + \frac{1}{2} \cdot 3 \\ -1 &= b. \end{aligned}$$

Thus, the second line is $y = -1 + \frac{1}{2}x$, and it is for the part of the function where $x \geq 3$.

Therefore, the function is:

$$y = \begin{cases} 5 - x & \text{for } x < 3 \\ -1 + \frac{1}{2}x & \text{for } x \geq 3. \end{cases}$$

6. We find the formulas for each of the lines. For the first, we use the two points we have, $(1, 6.5)$ and $(3, 5.5)$. We find the slope: $(5.5 - 6.5)/(3 - 1) = -\frac{1}{2}$. Using the slope of $-\frac{1}{2}$, we solve for the y-intercept:

$$\begin{aligned} 6.5 &= b - \frac{1}{2} \cdot 1 \\ 7 &= b. \end{aligned}$$

Thus, the first line is $y = 7 - \frac{1}{2}x$, and it is for the part of the function where $x \leq 3$.

We follow the same method for the second line, using the points $(3, 2)$ and $(5, 2)$. Noting that the y values are the same, we know the slope is zero and that the line is $y = 2$ for the part of the function where $3 < x \leq 5$.

We follow the same method for the third line, using the points $(5, 7)$ and $(7, 3)$. We find the slope: $(3-7)/(7-5) = -2$. Using the slope of -2, we solve for the y-intercept:

$$\begin{aligned} 3 &= b - 2 \cdot 7 \\ 17 &= b. \end{aligned}$$

Thus, the second line is $y = 17 - 2x$, and it is for the part of the function where $x > 5$.

Therefore, the function is:

$$y = \begin{cases} 7 - \frac{1}{2}x & \text{for } x \leq 3 \\ 2 & \text{for } 3 < x \leq 5 \\ 17 - 2x & \text{for } x > 5. \end{cases}$$

7. We find the formulas for each of the lines. For the first, we use the two points we have, $(1, 3.5)$ and $(3, 2.5)$. We find the slope: $(2.5 - 3.5)/(3 - 1) = -\frac{1}{2}$. Using the slope of $-\frac{1}{2}$, we solve for the y-intercept:

$$\begin{aligned} 3.5 &= b - \frac{1}{2} \cdot 1 \\ 4 &= b. \end{aligned}$$

Thus, the first line is $y = 4 - \frac{1}{2}x$, and it is for the part of the function where $1 \leq x \leq 3$.

We follow the same method for the second line, using the points $(5, 1)$ and $(8, 7)$. We find the slope: $(7-1)/(8-5) = 2$. Using the slope of 2, we solve for the y-intercept:

$$\begin{aligned} 1 &= b + 2 \cdot 5 \\ -9 &= b. \end{aligned}$$

Thus, the second line is $y = -9 + 2x$, and it is for the part of the function where $5 \leq x \leq 8$.

Therefore, the function is:

$$y = \begin{cases} 4 - \frac{1}{2}x & \text{for } 1 \leq x \leq 3 \\ -9 + 2x & \text{for } 5 \leq x \leq 8. \end{cases}$$

8. We find the formulas for each of the lines. For the first, we use the two points we have, $(1, 4)$ and $(3, 2)$. We find the slope: $(2 - 4)/(3 - 1) = -1$. Using the slope of -1, we solve for the y-intercept:

$$\begin{aligned} 4 &= b - 1 \cdot 1 \\ 5 &= b. \end{aligned}$$

Thus, the first line is $y = 5 - x$, and it is for the part of the function where $x \leq 3$.

We follow the same method for the second line, using the points $(3, 2)$ and $(5, 4)$. We find the slope: $(4-2)/(5-3) = 1$. Using the slope of 1, we solve for the y-intercept:

$$\begin{aligned} 4 &= b + 1 \cdot 5 \\ -1 &= b. \end{aligned}$$

Thus, the second line is $y = -1 + x$, and it is for the part of the function where $x \geq 3$.

Therefore, the function is:

$$y = \begin{cases} 5 - x & \text{for } x \leq 3 \\ -1 + x & \text{for } x \geq 3. \end{cases}$$

Notice that the value of $y = 2$ at $x = 3$ can be obtained from either formula.

Alternatively, this is the graph of the absolute value function,

$$y = |x - 3| + 2.$$

9. Since $G(x)$ is defined for all x, the domain is all real numbers. For $x < -1$ the values of the function are all negative numbers. For $-1 \geq x \geq 0$ the function's values are $4 \geq G(x) \geq 3$, while for $x > 0$ we see that $G(x) \geq 3$ and the values increase to infinity. The range is $G(x) < 0$ and $G(x) \geq 3$.

10. Since $F(x)$ is defined for all x, the domain is all real numbers. For $x \leq 1$ the values of the function are $-\infty \leq F(x) \leq 1$, while for $x > 1$ we see that $0 < F(x) < 1$, so the range is $F(x) \leq 1$.

11. We want to find all numbers x such that $|x| = 5$, or $|x - 0| = 5$. That is, we want the distance between x and 0 to be 5. Thus, x must be five units to the left or five units to the right of 0; that is, $x = -5$ and $x = 5$.

12. We want to find all numbers y such that $2|y| - 20 = 0$. Adding 20 to both sides and then dividing by 2, this is equivalent to $|y| = 10$ which we can rewrite as $|y - 0| = 10$. This means we want the distance between y and 0 to be 10. Thus, y must be ten units to the left or ten units to the right of 0; that is, $y = -10$ and $y = 10$.

13. We want to find all numbers x such that $|x - 3| = 7$. That is, we want the distance between x and 3 to be 7. Thus, x must be seven units to the left or seven units to the right of 3; that is, $x = -4$ and $x = 10$.

14. We want to find all numbers x such that $20 = 10|x - 5|$. Dividing both sides by 10, this is equivalent to all all numbers x such that $2 = |x - 5|$. That is, we want the distance between x and 5 to be 2. Thus, x must be two units to the left or two units to the right of 5; that is, $x = 3$ and $x = 7$.

15. We want to find all numbers x such that $|y-3| = 8 - |y-3|$. Subtracting $|y-3|$ from both sides, this is equivalent to $2|y-3| = 8$. Then, dividing both sides by 2, this is equivalent to $|y-3| = 4$. That is, we want the distance between y and 3 to be 4. Thus, x must be four units to the left or four units to the right of 3; that is, $x = -1$ and $x = 7$.

16. We want to find all numbers y such that $|2y-6| = 8$. That is, we want the distance between $2y$ and 6 to be 8. Thus, $2y$ must be eight units to the left or eight units to the right of 6; that is,

$$2y = -2 \qquad \text{or} \qquad 2y = 14.$$

In the first case, $y = -1$; in the second, $y = 7$. So the values of y such that $|2y-6| = 8$ are $y = -1$ and $y = 7$.

Problems

17. **(a)** Yes, because every value of x is associated with exactly one value of y.
(b) No, because some values of y are associated with more than one value of x.
(c) Only part (a) leads to a function. Its range is $y = 1, 2, 3, 4$.

18. **(a)** Figures 2.24 and 2.23 show the two functions x and $\sqrt{x^2}$. Because the two functions do not coincide for $x < 0$, they cannot be equal. The graph of $\sqrt{x^2}$ looks like the graph of $|x|$.

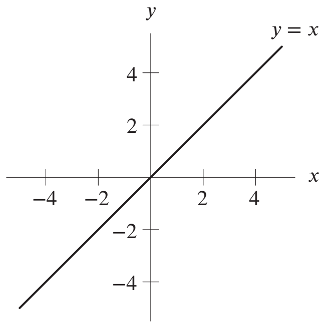

Figure 2.23

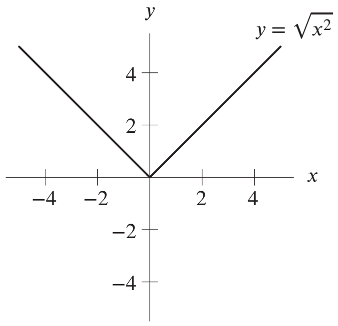

Figure 2.24

(b) Table 2.4 is the complete table. Because the two functions do not coincide for $x < 0$ they cannot be equal. The table for $\sqrt{x^2}$ looks like a table for of $|x|$.

Table 2.4

x	−5	−4	−3	−2	−1	0	1	2	3	4	5
$\sqrt{x^2}$	5	4	3	2	1	0	1	2	3	4	5

(c) If $x > 0$, then $\sqrt{x^2} = x$, whereas if $x < 0$ then $\sqrt{x^2} = -x$. This is the definition of $|x|$. Thus we have shown $\sqrt{x^2} = |x|$.
(d) We see nothing because $\sqrt{x^2} - |x| = 0$, and the graphing calculator or computer has drawn a horizontal line on top of the x-axis.

19. **(a)** Figure 2.25 shows the function $u(x)$. Some graphing calculators or computers may show a near-vertical line close to the origin. The function seems to be -1 when $x < 0$ and 1 when $x > 0$.

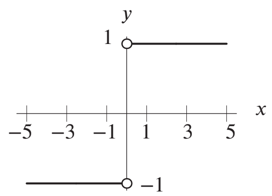

Figure 2.25

(b) Table 2.5 is the completed table. It agrees with what we found in part (a). The function is undefined at $x = 0$.

Table 2.5

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$\lvert x\rvert/x$	-1	-1	-1	-1	-1		1	1	1	1	1

(c) The domain is all x except $x = 0$. The range is -1 and 1.

(d) $u(0)$ is undefined, not 0. The claim is false.

20. (a) Since $-2 < 0$, we choose the formula $g(x) = -1$, getting $g(-2) = -1$. Since $2 \geq 0$, we choose the formula $g(x) = x^3$, getting $g(2) = 2^3 = 8$. The value $x = 0$ is the place on the x-axis, where we switch from one formula to the other. But the equal sign is attached to the formula for $g(x) = x^3$, so $g(0) = 0^3 = 0$.

(b) By graphing g we can see in Figure 2.26 that the domain includes the entire x-axis. On the vertical axis the only negative value is -1, so the range is $g(x) \geq 0$ and $g(x) = -1$.

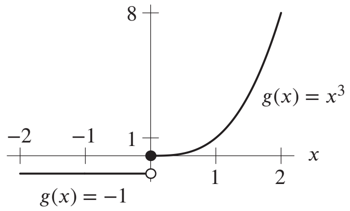

Figure 2.26

21. (a) Since zero lies in the interval $-1 \leq x \leq 1$, we find the function value from the formula $f(x) = 3x$. This gives $f(0) = 3 \cdot 0 = 0$. To find $f(3)$, we first note that $x = 3$ lies in the interval $1 < x \leq 5$, so we find the function value from the formula $f(x) = -x + 4$. The result is $f(3) = -3 + 4 = 1$.

(b) By graphing f we can see in Figure 2.27 that the combined domain is $-1 \leq x \leq 5$ and the range is $-3 \leq f(x) \leq 3$.

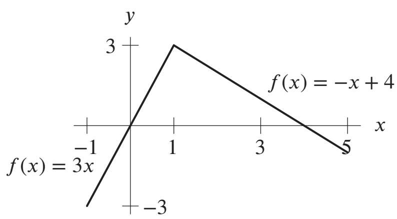

Figure 2.27

22. (a) We have $f(-2) = \dfrac{1}{-2} = -\dfrac{1}{2}$ and $f(2) = \sqrt{2}$.

(b) For $x \geq -1$, we have $f(x) \geq 0$, because $\sqrt{x}$ and x^2 are always non-negative. For $x < -1$, we see $1/x$ is between -1 and 0, thus the range is $f(x) > -1$.

23. The statement

$y = x^2$ for x less than zero, and $y = x - 1$ for x greater than or equal to zero

can be condensed to $y = \begin{cases} x^2 & \text{for} \quad x < 0 \\ x - 1 & \text{for} \quad x \geq 0 \end{cases}$

24. (a) Let $y = f(x)$ be the cost of a stripping and refinishing job for a floor which is x square feet in area. When the area is less than or equal to 150 square feet, the price is \$1.83 times the number of square feet. Thus, for x-values up through 150, we have $f(x) = 1.83x$. However, if the area is more than 150 square feet, the extra cost of toxic waste disposal is added, giving $f(x) = 1.83x + 350$. The maximum total area for a job is 1000 square feet, so the formula is

$$f(x) = \begin{cases} 1.83x, & 0 \leq x \leq 150 \\ 1.83x + 350, & 150 < x \leq 1000 \end{cases}$$

(b) The graph is in Figure 2.28. Note that when $x = 150$ sq ft, $y = 1.83(150) = \$274.5$. When x goes above 150 sq ft, the cost jumps by \$350 to \$624.5.

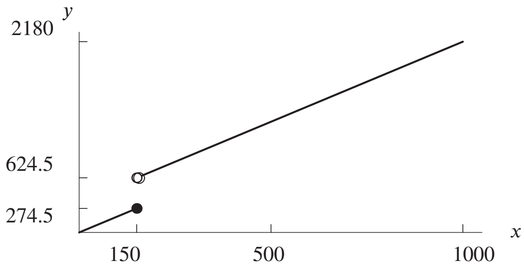

Figure 2.28

No floor has a negative area and the company will refinish any floor whose area is less than or equal to 1000 square feet, so

$$\text{Domain is} \quad 0 \le x \le 1000.$$

As the size of the floor gets bigger, the cost increases. The smallest value of the range occurs when $x = 0$ and the largest value occurs when $x = 1000$. So the smallest value is $f(0) = 0$ and the largest is $f(1000) = 2180$. There is a gap, though, in the values of the range. The value of $f(x)$ jumps from 274.5, when $x = 150$, to more than 624.5 when x is just slightly more than 150. Putting all these pieces together, we have

$$\text{Range is} \quad 0 \le y \le 274.5 \text{ or } 624.5 < y \le 2180.$$

25. **(a)** Upon entry, the cost is \$2.50. The tax surcharge of \$0.50 is added to the fare. So, the initial cost will be \$3.00. The cost for the first $1/5$ mile adds \$0.40, giving a fare of \$3.40. For a journey of $2/5$ mile, another \$0.40 is added for a fare of \$3.80. Each additional $1/5$ mile gives an another increment of \$0.40. See Table 2.6.

Table 2.6

Miles	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
Cost	3.00	3.40	3.80	4.20	4.60	5.00	5.40	5.80	6.20	6.60	7.00

(b) The table shows that the cost for a 1.2 mile trip is \$5.40.
(c) From the table, the maximum distance one can travel for 5.80 is 1.4 mile.
(d) See Figure 2.29.

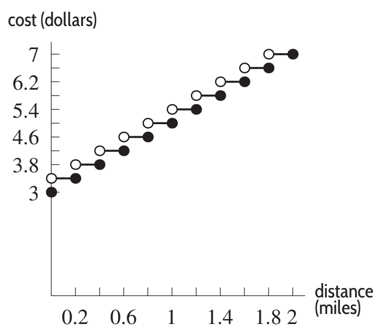

Figure 2.29

26. **(a)** The dots in Figure 2.30 represent the graph of the function.

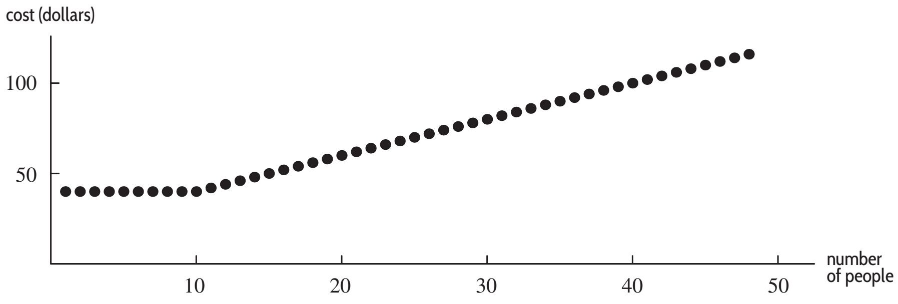

Figure 2.30

(b) Since admission is charged for whole numbers of people between 1 and 50, the domain is the integers from 1 to 50. The minimum cost is \$40. The maximum occurs for 50 people and is $\$40 + 40(\$2) = \$120$. Since the lowest cost is \$40, and each additional person costs \$2, the range only includes numbers which are multiples of 2. Thus, the range is all the even integers from 40 to 120.

27. **(a)** The smaller the difference, the smaller the refund. The smallest possible difference is \$0.01. This translates into a refund of $\$1.00 + \$0.01 = \$1.01$.

(b) Looking at the refund rules, we see that there are three separate cases to consider. The first case is when 10 times the difference is less than \$1. If the difference is more than 0 but less than 10¢, you will receive \$1 plus the difference. The formula for this is:

$$y = 1 + x \quad \text{for} \quad 0 < x < 0.10.$$

In the second case, 10 times the difference is between \$1 and \$5. This will be true if the difference is between 10¢ and 50¢. The formula for this is:

$$y = 10x + x \quad \text{for} \quad 0.10 \le x \le 0.50.$$

In the third case, 10 times the difference is more than \$5. If the difference is more than 50¢, then you receive \$5 plus the difference or:

$$y = 5 + x \quad \text{for} \quad x > 0.50.$$

Putting these cases together, we get:

$$y = \begin{cases} 1 + x & \text{for } 0 < x < 0.1 \\ 10x + x & \text{for } 0.1 \le x \le 0.5 \\ 5 + x & \text{for } x > 0.5. \end{cases}$$

(c) We want x such that $y = 9$. Since the highest possible value of y for the first case occurs when $x = 0.09$, and $y = 1 + 0.09 = \$1.09$, the range for this case does not go high enough. The highest possible value for the second case occurs when $x = 0.5$, and $y = 10(0.5) + 0.5 = \$5.50$. This range is also not high enough. So we look to the third case where $x > 0.5$ and $y = 5 + x$. Solving $5 + x = 9$, we find $x = 4$. So the price difference would have to be \$4.

(d) See Figure 2.31.

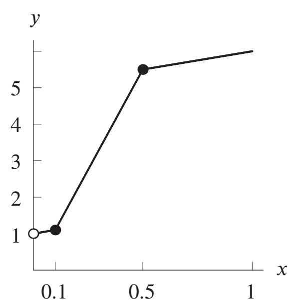

Figure 2.31

28. **(a)** See Table 2.7. Note that the cost is in fractions of a penny, so answers are rounded to the nearest penny.

Table 2.7

kWh	0	5	10	15	20	25	30	35	40
Cost in dollars	0.16	0.39	0.62	1.16	1.69	2.23	2.77	3.30	3.84

(b) In dollars, $C(n) = \begin{cases} 0.1570 + 0.0466n, & 0 \le n \le 10 \\ 0.6230 + 0.1071(n - 10), & n > 10 \end{cases}$

(c) The formula gives a cost of \$3.09 for 33 kWh usage.

(d) Since 33 kWh cost \$3.09, we use the formula for $n > 10$ and solve

$$3 = 0.6230 + 0.1071(n - 10),$$

giving $n = 32.194$.

29. **(a)** Figure 2.32 shows the rates for the first and last periods of the year. Figure 2.34 shows the rates for holiday periods (Dec 25–Jan 3, Jan 16–18, Feb 3–21) and Figure 2.33 shows the rates for the other times.

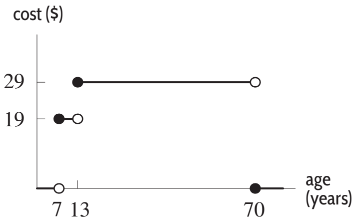

Figure 2.32: Opening/Closing

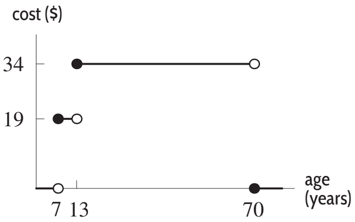

Figure 2.33: Regular rates

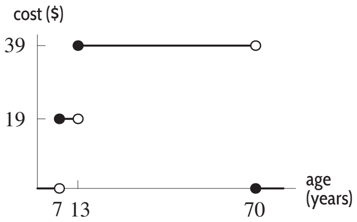

Figure 2.34: Holiday rates

(b) Ages 13-69.

(c) See Figure 2.35.

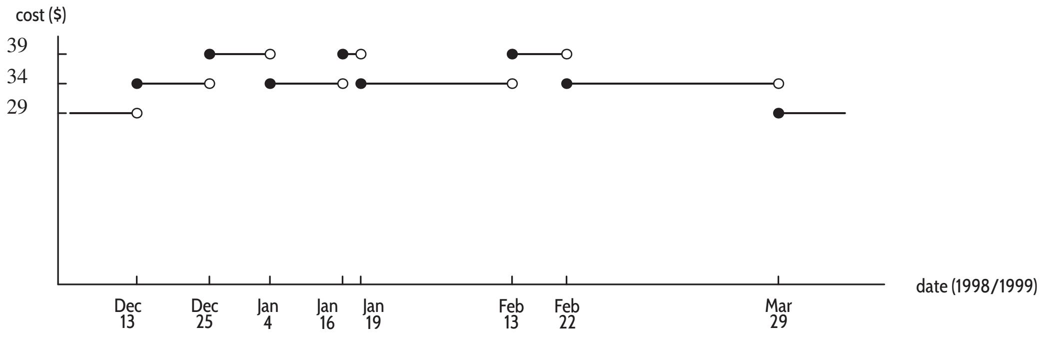

Figure 2.35: Cost as a function of date for 13-69 years old

(d) Rates through 12 December and after 19 March represent early-season and late-season rates, respectively; these are off-peak rates, since it makes economic sense to cut rates when there are fewer skiers. Holiday rates took effect from 25 December through 3 January because of the Christmas/New Year's holiday; they took effect from 16 January through 18 January for Martin Luther King's holiday; they took effect from 13 February through 21 February for the Presidents' Week holiday; it makes economic sense to charge peak rates during the holidays, as more skiers are available to use the facility. Other times represent rates during the heart of the winter skiing season; these are the regular rates.

30. **(a)** We have $f(x) = \begin{cases} x^2 - 4 & \text{for } x^2 - 4 \geq 0 \\ -(x^2 - 4) & \text{for } x^2 - 4 < 0 \end{cases}$

or

$$f(x) = \begin{cases} x^2 - 4 & \text{for } x \leq -2 \\ 4 - x^2 & \text{for } -2 < x < 2 \\ x^2 - 4 & \text{for } x \geq 2 \end{cases}.$$

(b) See Figure 2.36.

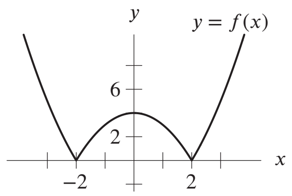

Figure 2.36

31. **(a)** We have $f(x) = \begin{cases} 2x - 6 & \text{for } 2x - 6 \geq 0 \\ -(2x - 6) & \text{for } 2x - 6 < 0 \end{cases}$

or

$$f(x) = \begin{cases} 2x - 6 & \text{for } x \geq 3 \\ 6 - 2x & \text{for } x < 3 \end{cases}.$$

(b) See Figure 2.37.

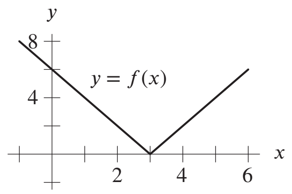

Figure 2.37

Solutions for Section 2.4

Skill Refresher

S1. Substituting $x = 4$ into $f(x)$, we have $f(4) = \sqrt{4} = 2$.

S2. Substituting $x = 4$ into $g(x)$, we have $g(4) = \sqrt{4} + 6 = 8$. Notice that $g(x)$ is a vertical shift of $f(x) = \sqrt{x}$ up 6 units, and thus the point $(4, 2)$ on the graph of $f(x)$ has been shifted to the point $(4, 8)$ on the graph of $g(x)$.

S3. Substituting $x = 4$ into $h(x)$, we have $h(4) = \sqrt{4} - 3 = -1$. Notice that $h(x)$ is a vertical shift of $f(x) = \sqrt{x}$ down 3 units, and thus the point $(4, 2)$ on the graph of $f(x)$ has been shifted to the point $(4, -1)$ on the graph of $h(x)$.

S4. Substituting $x = 4$ into $k(x)$, we have $k(4) = \sqrt{4 + 5} = \sqrt{9} = 3$. Notice that $k(x)$ is a horizontal shift of $f(x) = \sqrt{x}$ to the left by 5 units.

S5. The solutions are

$$x = -2 \quad \text{and} \quad x = 2.$$

S6. The equation is $x^2 = 9$, so solutions are $x = -3$ and $x = 3$.

S7. The solutions are $x - 1 = -2$ and $x - 1 = 2$, giving

$$x = -1 \qquad \text{and} \qquad x = 3.$$

S8. The solutions are $x + 1 = -2$ and $x + 1 = 2$, giving

$$x = -3 \qquad \text{and} \qquad x = 1.$$

Exercises

1. **(a)**

x	-1	0	1	2	3
$g(x)$	-3	0	2	1	-1

The graph of $g(x)$ is shifted one unit to the right of $f(x)$.

(b)

x	-3	-2	-1	0	1
$h(x)$	-3	0	2	1	-1

The graph of $h(x)$ is shifted one unit to the left of $f(x)$.

(c)

x	-2	-1	0	1	2
$k(x)$	0	3	5	4	2

The graph $k(x)$ is shifted up three units from $f(x)$.

(d)

x	-1	0	1	2	3
$m(x)$	0	3	5	4	2

The graph $m(x)$ is shifted one unit to the right and three units up from $f(x)$.

2. The graph of $y = f(x + 3) + 3$ is the graph of f shifted left by 3 units and up by 3 units. See Figure 2.38.

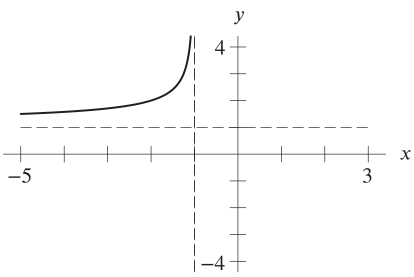

Figure 2.38: Graph of $y = f(x + 3) + 3$

3. See Figure 2.39.

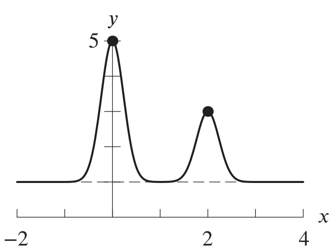

Figure 2.39

4. See Figure 2.40.

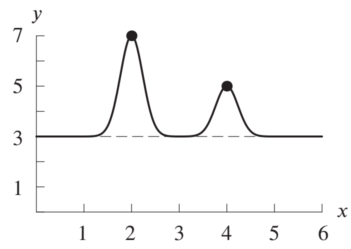

Figure 2.40

5. See Figure 2.41.

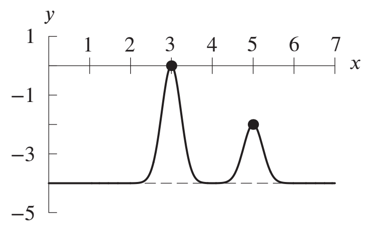

Figure 2.41

6. See Figure 2.42.

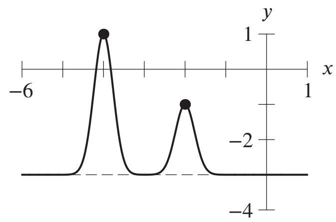

Figure 2.42

7. **(a)** The translation $f(x)+5$ moves the graph up 5 units. The x-coordinate is not changed, but the y-coordinate is $-4+5 = 1$. The new point is $(3, 1)$.

(b) The translation $f(x + 5)$ shifts the graph to the left 5 units. The y-coordinate is not changed, but the x-coordinate is $3 - 5 = -2$. The new point is $(-2, -4)$.

(c) This translation shifts both the x and y coordinates, 3 units right and 2 units down, resulting in $(6, -6)$.

8. The translation shifts the graph to the right 2 units, so the new domain is $0 < x < 9$.

9. The range shifts the graph down 150 units, so the new range is $-50 \le R(s) - 150 \le 50$.

10. **(a)** (i) To evaluate $f(x)$ for $x = 6$, we find from the table the value of $f(x)$ corresponding to an x-value of 6. In this case, the corresponding value is 248. Thus, $f(x)$ at $x = 6$ is $f(6) = 248$.

(ii) The value of $f(5)$ is the value of $f(x)$ corresponding to $x = 5$, or 145. Thus, $f(5) - 3 = 145 - 3 = 142$.

(iii) Since $f(5-3)$ is the same thing as $f(2)$, the value of $f(x)$ corresponding to $x = 2$, we have $f(5-3) = f(2) = 4$.

(iv) Since $g(x) + 6$ for $x = 2$ equals $g(2) + 6$ and $g(2)$ is the value of $g(x)$ corresponding to an x-value of 2, we have $g(2) = 6$ and $g(2) + 6 = 6 + 6 = 12$.

(v) We have $g(x+6)$ for $x = 2$, which is $g(2+6) = g(8)$. Looking at the table, we see that $g(8) = 378$. Thus, $g(x+6)$ for $x = 2$ equals 378.
(vi) Since $g(x)$ for $x = 0$ equals $g(0) = -6$, we have $3 \cdot (g(0)) = 3 \cdot (-6) = -18$.
(vii) Since $f(3x)$ for $x = 2$ equals $f(3 \cdot 2) = f(6)$, we know from part (a) that $f(6) = 248$. Thus, $f(3x)$ for $x = 2$ equals 248.
(viii) Since $f(x)-f(2)$ for $x = 8$ equals $f(8)-f(2)$, we have $f(8) = 574$ and $f(2) = 4$, so $f(8)-f(2) = 574-4 = 570$.
(ix) Since $g(x+1) - g(x)$ for $x = 1$ equals $g(1+1) - g(1) = g(2) - g(1)$ and we have $g(2) = 6$ and $g(1) = -7$, we have $g(2) - g(1) = 6 - (-7) = 6 + 7 = 13$.

(b) (i) To find x such that $g(x) = 6$, we look for the entry in the table at which $g(x) = 6$ and then see what the corresponding x-value is. In this case, it is 2. Thus, $g(x) = 6$ for $x = 2$.
(ii) We use the same principle as that in part (i): $f(x) = 574$ when $x = 8$.
(iii) Again, this is just like part (i): $g(x) = 281$ when $x = 7$.

(c) Solving $x^3 + x^2 + x - 10 = 7x^2 - 8x - 6$ involves finding those values of x for which both sides of the equation are equal, or where $f(x) = g(x)$. Looking at the table, we see that $f(x) = g(x) = -7$ for $x = 1$, and $f(x) = g(x) = 74$ for $x = 4$.

Problems

11. The graph in Figure 2.30 is a result of shifting the function in Figure 2.29 right 2 units and down 6 units. Thus $y = f(x-2) - 6$ is a formula for the function in Figure 2.30.

12. **(a)** This graph is the graph of $m(r)$ shifted upwards by two units. Thus, the formula for $n(r)$ is

$$n(r) = m(r) + 2.$$

(b) This graph is the graph of $m(r)$ shifted to the right by one unit. Thus, the formula for $p(r)$ is

$$p(r) = m(r-1).$$

(c) This graph is the graph of $m(r)$ shifted to the left by 1.5 units. Thus, the formula for $k(r)$ is

$$k(r) = m(r+1.5).$$

(d) This graph is the graph of $m(r)$ shifted to the right by 0.5 units and downwards by 2.5 units. Thus, the formula for $w(r)$ is

$$w(r) = m(r-0.5) - 2.5.$$

13. **(a)** The translation should leave the x-coordinate unchanged, and shift the y-coordinate up 3; so $y = g(x) + 3$.
(b) The translation should leave the y-coordinate unchanged, and shift the x-coordinate right by 2; so $y = g(x-2)$.

14. **(a)** Notice that the value of $h(x)$ at every value of x is 2 less than the value of $f(x)$ at the same x value. Thus

$$h(x) = f(x) - 2.$$

(b) Observe that $g(0) = f(1)$, $g(1) = f(2)$, and so on. In general,

$$g(x) = f(x+1).$$

(c) The values of $i(x)$ are two less than the values of $g(x)$at the same x value. Thus

$$i(x) = f(x+1) - 2.$$

15. Notice that w reaches its maximum value of $y = 13$ at $x = 5$, whereas v reaches its maximum value of $y = 20$ at $x = 0$. Since we know that w is a transformation of v, this suggests that the graph of w is the graph of v shifted to the right by 5 units and down by 7 units:

$$w(x) = v(x-5) - 7.$$

Thus, $h = 5$ and $k = -7$. Checking our answer for $x = 3$ and $x = 4$, we see that

$$w(3) = v(3-5) - 7 = \underbrace{v(-2)}_{11} - 7 = 4$$
$$w(4) = v(4-5) - 7 = \underbrace{v(-1)}_{17} - 7 = 10,$$

and so on, as required.

16. At $t = 3$ months, Jonah's weight is

$$V = s(3) + 2.$$

Since $s(3)$ is the average weight of a 3-month old boy, we see that at 3 months, Jonah weighs 2 pounds more than average. Similarly, at $t = 6$ months we have

$$V = s(6) + 2,$$

which means that, at 6 months, Jonah weighs 2 pounds more than average. In general, Jonah weighs 2 pounds more than average for babies of his age.

17. Since $W = s(t + 4)$, at age $t = 3$ months Ben's weight is given by

$$W = s(3 + 4) = s(7).$$

We defined $s(7)$ to be the average weight of a 7-month old baby. At age 3 months, Ben's weight is the same as the average weight of 7-month old babies. Since, on average, a baby's weight increases as the baby grows, this means that Ben is heavier than the average for a 3-month old. Similarly, at age $t = 6$, Ben's weight is given by

$$W = s(6 + 4) = s(10).$$

Thus, at 6 months, Ben's weight is the same as the average weight of 10-month old babies. In both cases, we see that Ben is above average in weight.

18. **(a)** $P(t) + 100$ describes a population that is always 100 people larger than the original population.

(b) $P(t + 100)$ describes a population that has the same number of people as the original population, but the number occurs 100 years earlier.

19. Since the $+3$ is an outside change, this transformation shifts the entire graph of $q(z)$ up by 3 units. That is, for every z, the value of $q(z) + 3$ is three units greater than $q(z)$.

20. Since the $-a$ is an outside change, this transformation shifts the entire graph of $q(z)$ down by a units. That is, for every z, the value of $q(z) - a$ is a units less than $q(z)$.

21. Since this is an inside change, the graph is four units to the left of $q(z)$. That is, for any given z value, the value of $q(z+4)$ is the same as the value of the function q evaluated four units to the right of z (at $z + 4$).

22. Since this is an inside change, the graph is a units to the right of $q(z)$. That is, for any given z value, the value of $q(z - a)$ is the same as the value of the function q evaluated a units to the left of z (at $z - a$).

23. From the inside change, we know that the graph is shifted b units to the left. From the outside change, we know that it is shifted a units down. So, for any given z value, the graph of $q(z + b) - a$ is b units to the left and a units below the graph of $q(z)$.

24. From the inside change, we know that the graph is shifted $2b$ units to the right. From the outside change, we know that it is shifted ab units up. So, for any given z value, the graph of $q(z - 2b) + ab$ is $2b$ units to the right and ab units above the graph of $q(z)$.

25. **(a)** There are many possible graphs, but all should show seasonally related cycles of temperature increases and decreases, as in Figure 2.43.

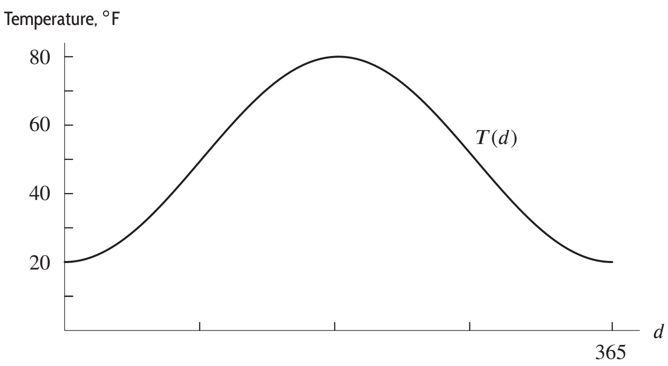

Figure 2.43

(b) While there are a wide variety of correct answers, the value of $T(6)$ is a temperature for a day in early January, $T(100)$ for a day in mid-April, and $T(215)$ for a day in early August. The value for $T(371) = T(365 + 6)$ should be close to that of $T(6)$.
(c) Since there are usually 365 days in a year, $T(d)$ and $T(d + 365)$ represent average temperatures on days which are a year apart.
(d) $T(d+365)$ is the average temperature on the same day of the year a year earlier. They should be about the same value. Therefore, the graph of $T(d + 365)$ should be about the same as that of $T(d)$.
(e) The graph of $T(d) + 365$ is a shift upward of $T(d)$, by 365 units. It has no significance in practical terms, other than to represent a temperature that is 365° hotter than the average temperature on day d.

26. **(a)** On day d, high tide in Tacoma, $T(d)$, is 1 foot higher than high tide in Seattle, $S(d)$. Thus, $T(d) = S(d) + 1$.
(b) On day d, height of the high tide in Astoria equals high tide of the previous day, i.e. $d - 1$, in Seattle. Thus, $A(d) = S(d - 1)$.

27. Subtract the normal temperature from the thermometer reading. The fever is

$$f(t) = H(t) - 37 \text{ degrees.}$$

Solutions for Section 2.5

Skill Refresher

S1. Adding 4 to both sides and dividing by 3, we get $y = \frac{x+4}{3}$.

S2. First we subtract $4x$ from both sides:

$$\begin{aligned} 4x - 3y - 4x &= 7 - 4x \\ -3y &= 7 - 4x \\ y &= \frac{7 - 4x}{-3}. \end{aligned}$$

S3. First we multiply both sides by $y - 2$ and get $x(y - 2) = 2y + 1$.

$$\begin{aligned} xy - 2x &= 2y + 1 \\ xy - 2y &= 2x + 1 \\ y(x - 2) &= 2x + 1 \\ y &= \frac{2x + 1}{x - 2}. \end{aligned}$$

S4. First we multiply both sides by $1 - y$ and get $x(1 - y) = 4y$.

$$\begin{aligned} x - xy &= 4y \\ -xy - 4y &= -x \\ y(-x - 4) &= -x \\ y &= \frac{-x}{-x - 4} \\ y &= \frac{x}{x + 4}. \end{aligned}$$

S5. Adding 2 to both sides, we get $\sqrt{y} = x + 2$. Squaring both sides, we get

$$y = (x + 2)^2 = x^2 + 4x + 4.$$

S6. Add 4 to both sides and take the cube root to obtain $y = \sqrt[3]{x+4}$.

S7. Add $(2y)^3$ to both sides and subtract x from both sides to get

$$\begin{aligned} (2y)^3 &= 5 - x \\ 2y &= \sqrt[3]{5-x} \\ y &= \frac{\sqrt[3]{5-x}}{2}. \end{aligned}$$

S8.

$$\begin{aligned} 5\left(\frac{1}{5}x - 1\right) + 5 &= x - 5 + 5 \\ &= x. \end{aligned}$$

S9.

$$4x^2 + 4x + 1 - 4 = 4x^2 + 4x - 3.$$

S10. We have

$$\begin{aligned} 3(y-2)^2 - 7 &= 3(y^2 - 4y + 4) - 7 \\ &= 3y^2 - 12y + 12 - 7 \\ &= 3y^2 - 12y + 5. \end{aligned}$$

S11. We have $(1-t)^2 - (1-t) = 1 - 2t + t^2 - 1 + t = t^2 - t$.

Exercises

1. The domain of $f^{-1}(y)$ is the range of $f(x)$, that is, all numbers between -3 and 2. Thus:

$$\text{Domain of } f^{-1}(y):\ -3 \leq \text{ all real numbers } \leq 2.$$

The range of $f^{-1}(y)$ is the domain of $f(x)$, that is all numbers between -2 and 3. Thus:

$$\text{Range of } f^{-1}(y):\ -2 \leq \text{ all real numbers } \leq 3.$$

2. The domain of $f^{-1}(y)$ is the range of $f(x)$, that is, all numbers between -6 and 4. Thus:

$$\text{Domain of } f^{-1}(y):\ -6 \leq \text{ all real numbers } \leq 4.$$

The range of $f^{-1}(y)$ is the domain of $f(x)$, that is all numbers between -6 and 9. Thus:

$$\text{Range of } f^{-1}(y):\ -6 \leq \text{ all real numbers } \leq 9.$$

3. The domain of $f^{-1}(y)$ is the range of $f(x)$, that is, all numbers between 1 and about 4.5. Thus, we estimate:

$$\text{Domain of } f^{-1}(y):\ 1 \leq \text{ all real numbers } \leq 4.5.$$

The range of $f^{-1}(y)$ is the domain of $f(x)$, that is all numbers between 0 and 6. Thus:

$$\text{Range of } f^{-1}(y):\ 0 \leq \text{ all real numbers } \leq 6.$$

4. The domain of $f^{-1}(y)$ is the range of $f(x)$, that is, all numbers between 0 and about 5.2. Thus:

$$\text{Domain of } f^{-1}(y):\ 0 \leq \text{ all real numbers } \leq 5.2.$$

The range of $f^{-1}(y)$ is the domain of $f(x)$, that is all numbers between 1 and 6. Thus:

$$\text{Range of } f^{-1}(y):\ 1 \leq \text{ all real numbers } \leq 6.$$

5. $f(g(0)) = f(1 - 0^2) = f(1) = 3 \cdot 1 - 1 = 2.$

6. $g(f(0)) = g(3 \cdot 0 - 1) = g(-1) = 1 - (-1)^2 = 0.$

7. $g(f(2)) = g(3 \cdot 2 - 1) = g(5) = 1 - (5)^2 = -24.$

8. $f(g(2)) = f(1 - 2^2) = f(-3) = 3(-3) - 1 = -10.$

9. $f(g(x)) = f(1 - x^2) = 3(1 - x^2) - 1 = 2 - 3x^2.$

10. $g(f(x)) = g(3x - 1) = 1 - (3x - 1)^2 = 1 - (9x^2 - 6x + 1) = -9x^2 + 6x.$

11. $f(f(x)) = f(3x - 1) = 3(3x - 1) - 1 = 9x - 4.$

12. $g(g(x)) = g(1 - x^2) = 1 - (1 - x^2)^2 = 2x^2 - x^4.$

13. $A(f(t))$ is the area, in square centimeters, of the circle at time t minutes.

14. $R(f(p))$ is the revenue, in millions of dollars, when the price of oil is p dollars/barrel.

15. $C(A(d))$ is the price of a pizza with diameter d.

16. The inverse function, $f^{-1}(P)$, gives the year in which population is P million. Units of $f^{-1}(P)$ are years.

17. The inverse function, $f^{-1}(T)$, gives the temperature in °F needed if the cake is to bake in T minutes. Units of $f^{-1}(T)$ are °F.

18. The inverse function, $f^{-1}(N)$, is the number of days for N inches of snow to fall. Units of $f^{-1}(N)$ are days.

19. The inverse function, $C^{-1}(x)$, gives the weight in ounces of x calories of almonds.

20. The inverse function, $P^{-1}(c)$, gives the diameter in inches of a pizza costing c dollars.

21. Since $y = 2t + 3$, solving for t gives

$$\begin{aligned} 2t + 3 &= y \\ t &= \frac{y-3}{2} \\ f^{-1}(y) &= \frac{y-3}{2}. \end{aligned}$$

22. Since $y = 3x - 7$, solving for x gives

$$\begin{aligned} 3x - 7 &= y \\ x &= \frac{y+7}{3} \\ f^{-1}(y) &= \frac{y+7}{3}. \end{aligned}$$

23. Since $y = x^3 + 1$, solving for x gives

$$\begin{aligned} x^3 + 1 &= y \\ x^3 &= y - 1 \\ x &= (y-1)^{1/3} \\ f^{-1}(y) &= (y-1)^{1/3}. \end{aligned}$$

24. Since $Q = x^3 + 3$, solving for x gives

$$\begin{aligned} x^3 + 3 &= Q \\ x^3 &= Q - 3 \\ x &= (Q-3)^{1/3} \\ f^{-1}(Q) &= (Q-3)^{1/3} \end{aligned}$$

25. Since $A = \pi r^2$, solving for r gives

$$\begin{aligned} \frac{A}{\pi} &= r^2 \\ \sqrt{\frac{A}{\pi}} &= r \\ r = f^{-1}(A) &= \sqrt{\frac{A}{\pi}}. \end{aligned}$$

26. Since $y = 1 + \frac{1}{s}$, solving for s gives

$$\begin{aligned} y - 1 &= \frac{1}{s} \\ \frac{1}{y-1} &= s \\ g^{-1}(y) &= \frac{1}{y-1}. \end{aligned}$$

27. Since $P = \frac{3x}{2x+1}$, solving for x gives

$$\begin{aligned} \frac{3x}{2x+1} &= P \\ 3x &= 2xP + P \\ 3x - 2xP &= P \\ x(3 - 2P) &= P \\ x &= \frac{P}{3-2P} \\ .f^{-1}(P) &= \frac{P}{3-2P} \end{aligned}$$

.

28. We have

$$f(4) = 5 \cdot 4 = 20.$$

To find $f^{-1}(15)$, we find the value of x giving $f(x) = 15$. If $15 = 5x$, then $x = 15/5 = 3$. Thus, $f^{-1}(15) = 3$.

29. We have

$$k(3) = 4(3) - 10 = 12 - 10 = 2.$$

To find $k^{-1}(14)$, we find the value of x so that $k(x) = 14$. If $14 = 4x - 10$, then $24 = 4x$ so $x = 6$. Thus, $k^{-1}(14) = 6$.

30. We have

$$h\left(-\frac{1}{2}\right) = \frac{-1/2}{4} + 2 = -\frac{1}{8} + 2 = \frac{15}{8}.$$

If $8 = x/4 + 2$, then $x/4 = 6$, so $x = 24$. Thus, $h^{-1}(8) = 24$.

31. We have

$$g\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - 1 = \frac{23}{4}$$

If $-17 = 2x^3 - 1$, then $x^3 = -8$, so $x = -2$. Thus, $g^{-1}(-17) = -2$.

32. **(a)** Since the vertical intercept of the graph of f is $(0, 1.5)$, we have $f(0) = 1.5$.

(b) Since the horizontal intercept of the graph of f is $(2.2, 0)$, we have $f(2.2) = 0$.

(c) The function f^{-1} goes from y-values to x-values, so to evaluate $f^{-1}(0)$, we want the x-value corresponding to $y = 0$. This is $x = 2.2$, so $f^{-1}(0) = 2.2$.

(d) Solving $f^{-1}(?) = 0$ means finding the y-value corresponding to $x = 0$. This is $y = 1.5$, so $f^{-1}(1.5) = 0$.

33. **(a)** Since the vertical intercept of the graph of f is $(0, b)$, we have $f(0) = b$.
(b) Since the horizontal intercept of the graph of f is $(a, 0)$, we have $f(a) = 0$.
(c) The function f^{-1} goes from y-values to x-values, so to evaluate $f^{-1}(0)$, we want the x-value corresponding to $y = 0$. This is $x = a$, so $f^{-1}(0) = a$.
(d) Solving $f^{-1}(?) = 0$ means finding the y-value corresponding to $x = 0$. This is $y = b$, so $f^{-1}(b) = 0$.

Problems

34. We solve the equation $C = g(x) = 600 + 45x$ for x. Subtract 600 from both sides and divide both sides by 45 to get

$$x = \frac{1}{45}(C - 600).$$

So

$$g^{-1}(C) = \frac{1}{45}(C - 600).$$

35. Since

$$n = \frac{A}{250},$$

solving for A gives

$$A = 250n.$$

Thus, $A = f^{-1}(n) = 250n$.

36. Since $n = f(A)$, in $f(100)$ we have $A = 100$ ft^2. Evaluating $f(100)$ tells us how much paint is needed for 100 ft^2. Since

$$n = f(100) = \frac{100}{250} = 0.4,$$

it takes 0.4 gallon of paint to cover 100 ft^2.

In $f^{-1}(100)$, the 100 is the number of gallons, so $f^{-1}(100)$ represents the area which can be painted by 100 gallons:

$$A = f^{-1}(100) = 250 \cdot 100 = 25{,}000 \text{ ft}^2.$$

37. Since $f(A) = A/250$ and $f^{-1}(n) = 250n$, we have

$$f^{-1}(f(A)) = f^{-1}\left(\frac{A}{250}\right) = 250\frac{A}{250} = A.$$
$$f(f^{-1}(n)) = f(250n) = \frac{250n}{250} = n.$$

To interpret these results, we use the fact that $f(A)$ gives the number of gallons of paint needed to cover an area A, and $f^{-1}(n)$ gives the area covered by n gallons. Thus $f^{-1}(f(A))$ gives the area which can be covered by $f(A)$ gallons; that is, A square feet. Similarly, $f(f^{-1}(n))$ gives the number of gallons needed for an area of $f^{-1}(n)$; that is, n gallons.

38. **(a)** Since m is the number of daily miles driven, the domain of $f(m)$ is all numbers greater than or equal to 0 and up to 500, the maximum number of miles we can drive in a day. This is also the range of of $f^{-1}(C)$. Thus:

$$\text{Range of } f^{-1}(C)\text{: } 0 \le \text{ all integer numbers } \le 500.$$

The domain of $f^{-1}(C)$ is the range of $f(m)$, or the cost of driving 0 or more daily miles after renting the car. The base rental cost is 32 dollars regardless of miles driven. We also pay, at most, $32 + 0.19(500) = 127$ dollars (the cost of driving 500 miles). Thus, the domain of $f^{-1}(C)$ is:

$$\text{Domain of } f^{-1}(C)\text{: } 32 \le \text{ all real numbers } \le 127.$$

(b) We solve the equation $C = f(m) = 32 + 0.19m$ for d. Subtract 32 from both sides and divide both sides by 0.19 to get

$$m = \frac{1}{0.19}(C - 32).$$

So

$$f^{-1}(C) = \frac{100}{19}(C - 32).$$

39. **(a)** To find values of f, read the table from top to bottom, so

(i) $f(0) = 2$ (ii) $f(1) = 0$.

To find values of f^{-1}, read the table in the opposite direction (from bottom to top), so

(iii) $f^{-1}(0) = 1$ (iv) $f^{-1}(2) = 0$.

(b) Since the values $(0, 2)$ are paired in the table, we know $f(0) = 2$ and $f^{-1}(2) = 0$. Thus, knowing the answer to (i) (namely, $f(0) = 2$) tells us the answer to (iv). Similarly, the answer to (ii), namely $f(1) = 0$, tells us that the values $(1, 0)$ are paired in the table, so $f^{-1}(0) = 1$ too.

40. **(a)** $j(h(4)) = h^{-1}(h(4)) = 4$
(b) We don't know $j(4)$
(c) $h(j(4)) = h(h^{-1}(4)) = 4$
(d) $j(2) = 4$
(e) We don't know $h^{-1}(-3)$
(f) $j^{-1}(-3) = 5$, since $j(5) = -3$
(g) We don't know $h(5)$
(h) $(h(-3))^{-1} = \left(j^{-1}(-3)\right)^{-1} = 5^{-1} = 1/5$
(i) We don't know $(h(2))^{-1}$

41. Since we can take the cube root of any number, the domain of $t(a)$ is all real numbers. Since the range of $t(a)$ is the domain of its inverse function, we first compute the inverse of $t(a)$. Let $t(a) = y$. Solving for a gives

$$\begin{aligned} y &= \sqrt[3]{a+1} \\ y^3 &= a+1 \\ a &= y^3 - 1 \\ t^{-1}(y) &= y^3 - 1. \end{aligned}$$

Since $y^3 - 1$ is defined for any y, the domain of $t^{-1}(y)$ is all real numbers, hence the range of $t(a)$ is all real numbers.

42. Since we can cube any number, the domain of $n(r)$ is all real numbers. To find the range, we find the inverse function. Let $y = n(r)$. Solving for r, we get

$$\begin{aligned} y &= r^3 + 2 \\ y - 2 &= r^3 \\ r &= (y-2)^{1/3} \\ n^{-1}(y) &= (y-2)^{1/3}. \end{aligned}$$

Since the domain of $n^{-1}(y)$ is all real numbers, the range of $n(r)$ is all real numbers.

43. To evaluate $m(x)$, we must have $x - 2 > 0$. So the domain of $m(x)$ is all real numbers > 2. To find the range of $m(x)$, we find the inverse function of $m(x)$. Let $y = m(x)$. Solving for x, we get

$$\begin{aligned} y &= \frac{1}{\sqrt{x-2}} \\ \sqrt{x-2} &= \frac{1}{y} \\ x - 2 &= \frac{1}{y^2} \\ x &= \frac{1}{y^2} + 2 \\ m^{-1}(y) &= \frac{1}{y^2} + 2. \end{aligned}$$

The formula works for any y except $y = 0$. We know that y must be positive, since $\sqrt{x-2}$ is positive, so the range of $m(x)$ is all real numbers > 0.

44. To evaluate $p(x)$, we must have $3 - x > 0$. So the domain of $p(x)$ is all real numbers < 3. To find the range of $p(x)$, we find the inverse function of $p(x)$. Let $y = p(x)$. Solving for x, we get

$$\begin{aligned} y &= \frac{1}{\sqrt{3-x}} \\ \sqrt{3-x} &= \frac{1}{y} \\ 3 - x &= \frac{1}{y^2} \\ x &= 3 - \frac{1}{y^2} \\ p^{-1}(y) &= 3 - \frac{1}{y^2}. \end{aligned}$$

The formula works for any y except $y = 0$. We know that y must be positive, since $\sqrt{3-x}$ is positive, so the range of $p(x)$ is all real numbers > 0.

45. **(a)** $G(13)$ is the output corresponding to the input of $t = 13$. So $G(13)$ represents the GDP thirteen years after 2000. This tells us that, in 2013, the gross domestic product was \$16,011.2 billion.
(b) The input to the G^{-1} function is billions of dollars, so its output is a time in years after 2000. Thus, $G^{-1}(13{,}776) = 7$ tells us that, seven years after 2000, the GDP was 13,776 billion dollars. Thus, the GDP was \$13,776 billion in 2007.

46. **(a)** The cost of producing 5000 loaves is \$653.
(b) $C^{-1}(80)$ is the number of loaves of bread that can be made for \$80, namely 0.62 thousand or 620.
(c) The solution is $q = 6.3$ thousand. It costs \$790 to make 6300 loaves.
(d) The solution is $x = 150$ dollars, so 1.2 thousand, or 1200, loaves can be made for \$150.

47. **(a)** $f(3) = 4 \cdot 3 = 12$ is the perimeter of a square of side 3.
(b) $f^{-1}(20)$ is the side of a square of perimeter 20. If $20 = 4s$, then $s = 5$, so $f^{-1}(20) = 5$.
(c) To find $f^{-1}(P)$, solve for s:

$$\begin{aligned} P &= 4s \\ s &= \frac{P}{4} \\ f^{-1}(P) &= \frac{P}{4}. \end{aligned}$$

48. **(a)** $f(10) = 100 + 0.2 \cdot 10 = 102$ thousand dollars, the cost of producing 10 kg of the chemical.
(b) $f^{-1}(200)$ is the quantity of the chemical which can be produced for 200 thousand dollars. Since

$$\begin{aligned} 200 &= 100 + 0.2q \\ 0.2q &= 100 \\ q &= \frac{100}{0.2} = 500 \text{ kg}, \end{aligned}$$

we have $f^{-1}(200) = 500$.
(c) To find $f^{-1}(C)$, solve for q:

$$\begin{aligned} C &= 100 + 0.2q \\ 0.2q &= C - 100 \\ q &= \frac{C}{0.2} - \frac{100}{0.2} = 5C - 500 \\ f^{-1}(C) &= 5C - 500. \end{aligned}$$

49. We can find the inverse function by solving for t in our equation:

$$\begin{aligned} H &= \frac{5}{9}(t - 32) \\ \frac{9}{5}H &= t - 32 \\ \frac{9}{5}H + 32 &= t. \end{aligned}$$

This function gives us the temperature in degrees Fahrenheit if we know the temperature in degrees Celsius.

50. **(a)** We substitute zero into the function, giving:

$$H = f(0) = \frac{5}{9}(0 - 32) = -\frac{160}{9} = -17.778.$$

This means that zero degrees Fahrenheit is about -18 degrees Celsius.

(b) In Exercise 49, we found the inverse function. Using it with $H = 0$, we have:

$$t = f^{-1}(0) = \frac{9}{5}0 + 32 = 32.$$

This means that zero degrees Celsius is equivalent to 32 degrees Fahrenheit (the temperature at which water freezes).

(c) We substitute 100 into the function, giving:

$$H = f(100) = \frac{5}{9}(100 - 32) = \frac{340}{9} = 37.778.$$

This means that 100 degrees Fahrenheit is about 38 degrees Celsius.

(d) In Exercise 49, we found the inverse function. Using it with $H = 100$, we have:

$$t = f^{-1}(100) = \frac{9}{5}100 + 32 = 212.$$

This means that 100 degrees Celsius is equivalent to 212 degrees Fahrenheit (the temperature at which water boils).

51. We have

$$H = f(g(n)) = f(68 + 10 \cdot 2^{-n}) = \frac{5}{9}(68 + 10 \cdot 2^{-n} - 32) = 20 + \frac{50}{9}2^{-n},$$

and $f(g(n))$ gives the temperature, H, in degrees Celsius after n hours.

52. Since

$$T = 2\pi\sqrt{\frac{l}{g}},$$

solving for l gives

$$T^2 = 4\pi^2\frac{l}{g}$$

$$l = \frac{gT^2}{4\pi^2}.$$

Thus,

$$f^{-1}(T) = \frac{gT^2}{4\pi^2}.$$

The function $f^{-1}(T)$ gives the length of a pendulum of period T.

53. **(a)** $A = f(r) = \pi r^2$

(b) $f(0) = 0$

(c) $f(r+1) = \pi(r+1)^2$. This is the area of a circle whose radius is 1 cm more than r.

(d) $f(r) + 1 = \pi r^2 + 1$. This is the area of a circle of radius r, plus 1 square centimeter more.

(e) Centimeters.

54. Since $V = \frac{4}{3}\pi r^3$ and $r = 50 - 2.5t$, substituting r into V gives

$$V = f(t) = \frac{4}{3}\pi(50 - 2.5t)^3.$$

55. **(a)** Since the pizza is circular, $f(d) = \pi(d/2)^2$.

(b) Since a package costs \$2.99 and covers 250 in^2, we know

$$\text{Cost of pepperoni for 1 in}^2 = \frac{2.99}{250} = 0.01196 \text{ dollars.}$$

Thus

$$\text{Cost of pepperoni for } A \text{ in}^2 = 0.01196A \text{ dollars,}$$

so

$$C = g(A) = 0.01196A.$$

(c) Substituting for $A = f(d) = \pi(d/2)^2$ into g gives

$$C = g(f(d)) = 0.01196\pi(d/2)^2.$$

The function $g(f(d))$ gives the cost in dollars of adding pepperoni to a pizza of diameter d inches.

(d) The area of an 11-inch pizza is $f(11) = \pi(11/2)^2 = 95.033$ in^2. The cost of adding a pepperoni topping to an 11-inch pizza is $g(11) = 0.01196 \cdot 95.033 = 1.14$ dollars.

56. Since the oil slick is circular, $A = \pi r^2$, so substituting $r = 2t - 0.1t^2$ into the formula for A gives

$$A = f(t) = \pi(2t - 0.1t^2)^2.$$

57. **(a)** $f(60) = 30$. A car traveling at 60 km/hr needs 30 meters to stop.
(b) $f(70)$ should be between $f(60) = 30$ and $f(80) = 50$, so we estimate 40 meters.
(c) $f^{-1}(70) = 100$ because $f(100) = 70$. A car that took 70 meters to stop was traveling at 100 km/hr.

58. **(a)** $f(2) = 2.80$ means that 2 pounds of apples cost \$2.80.
(b) $f(0.5) = 0.70$ means that $1/2$ pound of apples cost \$0.70.
(c) $f^{-1}(0.35) = 0.25$ means that \$0.35 buys $1/4$ pound of apples.
(d) $f^{-1}(7) = 5$ means that \$7 buys 5 pounds of apples.

59. **(a)** Since $t = 0$ represents 2004, we see that $0 \le t \le 4$ is the domain. The corresponding outputs are the range, $375 \le C(t) \le 383$.
(b) $C(4)$ is the concentration of carbon dioxide in the earth's atmosphere when $t = 4$, which is the year 2008. We know that $C(4) = 383$, since in 2008, the concentration of carbon dioxide in the earth's atmosphere was 383 ppm.
(c) $C^{-1}(381)$ is the number of years after 2004 when the concentration was 381 ppm. From the data given in the problem, the actual number of years cannot be determined. (Note: For your information, the concentration was 381 ppm in the year 2007.)

Solutions for Section 2.6

Exercises

1. To determine concavity, we calculate the rate of change:

$$\frac{\Delta f(x)}{\Delta x} = \frac{1.3 - 1.0}{1 - 0} = 0.3$$
$$\frac{\Delta f(x)}{\Delta x} = \frac{1.7 - 1.3}{3 - 1} = 0.2$$
$$\frac{\Delta f(x)}{\Delta x} = \frac{2.2 - 1.7}{6 - 3} \approx 0.167.$$

The rates of change are decreasing, so we expect the graph of $f(x)$ to be concave down.

2. To determine concavity we calculate the rate of change:

$$\frac{\Delta f(t)}{\Delta t} = \frac{10 - 20}{1 - 0} = -10.$$
$$\frac{\Delta f(t)}{\Delta t} = \frac{6 - 10}{2 - 1} = -4.$$
$$\frac{\Delta f(t)}{\Delta t} = \frac{3 - 6}{3 - 2} = -3.$$
$$\frac{\Delta f(t)}{\Delta t} = \frac{1 - 3}{4 - 3} = -2.$$

It appears that a graph of this function would be concave up, because the average rate of change becomes less negative as t increases.

3. The graph appears to be concave up, as its slope becomes less negative as x increases.

4. The graph appears to be concave down, as its slope becomes less positive as x increases.

5. The slope of $y = x^2$ is always increasing, so its graph is concave up. See Figure 2.44.

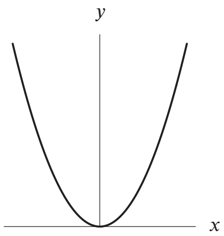

Figure 2.44

6. The slope of $y = -x^2$ is always decreasing, so its graph is concave down. See Figure 2.45.

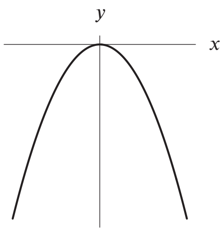

Figure 2.45

7. The slope of $y = x^3$ is always increasing on the interval $x > 0$, so its graph is concave up. See Figure 2.46.

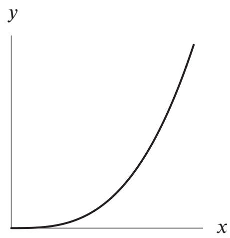

Figure 2.46

8. The slope of $y = x^3$ is always decreasing on the interval $x < 0$, so its graph is concave down. See Figure 2.47.

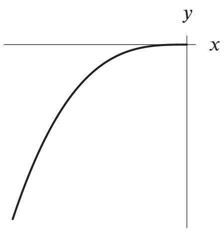

Figure 2.47

9. The rate of change between $x = 12$ and $x = 15$ is

$$\frac{\Delta H(x)}{\Delta x} = \frac{21.53 - 21.40}{15 - 12} \approx 0.043.$$

Similarly, we have

$$\frac{\Delta H(x)}{\Delta x} = \frac{21.75 - 21.53}{18 - 15} \approx 0.073$$
$$\frac{\Delta H(x)}{\Delta x} = \frac{22.02 - 21.75}{21 - 18} \approx 0.090.$$

The rate of change is increasing, so we expect the graph of $H(x)$ to be concave up.

10. The rate of change between $t = 1.5$ and $t = 2.4$ is

$$\frac{\Delta R(t)}{\Delta t} = \frac{-3.1 - (-5.7)}{2.4 - 1.5} = 2.889.$$

Similarly, we have

$$\frac{\Delta R(t)}{\Delta t} = \frac{-1.4 - (-3.1)}{3.6 - 2.4} = 1.417$$
$$\frac{\Delta R(t)}{\Delta t} = \frac{0 - (-1.4)}{4.8 - 3.6} = 1.167.$$

The rate of change is decreasing, so we expect the graph to be concave down.

11. A possible graph is in Figure 2.48.

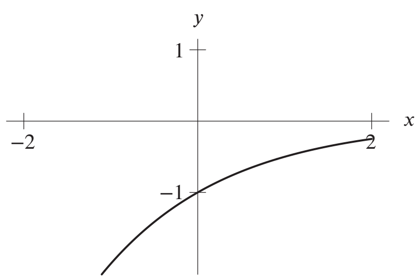

Figure 2.48

12. A possible graph is in Figure 2.49.

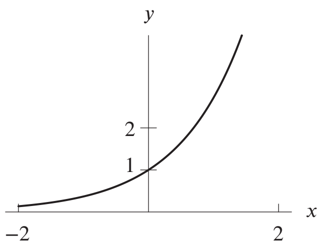

Figure 2.49

Problems

13. The graph is increasing for all x. It is concave down for $x < o$ and concave up for $x > 0$. Thus we have

(a) Increasing and concave down $x < 0$
(b) Decreasing and concave down nowhere
(c) Increasing and concave up $x > 0$
(d) Decreasing and concave down nowhere

14. The graph increasing for $-1 < x < 1$ and decreasing for $x < -1$ and $x > 1$. The graph is concave up for $x < 0$ and concave down for $x > 0$.

(a) Increasing and concave down $0 < x < 1$
(b) Decreasing and concave down $x > 1$
(c) Increasing and concave up $-1 < x < 0$
(d) Decreasing and concave up $x < -1$

15. The graph is decreasing for all $x > 0$ and $x < 0$.. The graph is concave up for $x > 0$ and concave down for $x < 0$. Thus we have

(a) Increasing and concave down nowhere
(b) Decreasing and concave down $x < 0$
(c) Increasing and concave up nowhere
(d) Decreasing and concave up $x > 0$

16. The graph is increasing for $x < 0$ and decreasing for $x > 0$. The graph is concave up for $x < -1$ and $x > 1$. It is concave down for $-1 < x < 1$. Thus we have

(a) Increasing and concave down $-1 < x < 0$
(b) Decreasing and concave $0 < x < 1$
(c) Increasing and concave up $x < -1$
(d) Decreasing and concave up $x > 1$

17. Since more and more of the drug is being injected into the body, this is an increasing function. However, since the rate of increase of the drug is slowing down, the graph is concave down.

18. This function is decreasing. As the coffee cools off, the temperature decreases at a slower rate. Since the rate of change is less negative, the graph is concave up.

19. The function is increasing throughout. At first, the graph is concave up. As more and more people hear the rumor, the rumor spreads more slowly, which means that the graph is then concave down.

20. This function is increasing throughout and the rate of increase is increasing, so the graph is concave up.

21. Since new people are always trying the product, it is an increasing function. At first, the graph is concave up. After many people start to use the product, the rate of increase slows down and the graph becomes concave down.

22. Many answers are possible. See Figure 2.50.

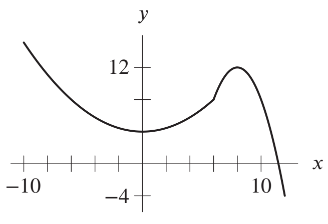

Figure 2.50

23. The graphical representation of the data is misleading because in the graph the number of violent crimes is put on the horizontal axis, which gives the graph the appearance of leveling out. This can fool us into believing that crime is leveling out. Note that it took from 1998 to 2000, about 2 years, for the number of violent crimes to go from 500 to 1,000, but it took less than 1/2 a year for that number to go from 1,500 to 2,000, and even less time for it to go from 2,000 to 2,500. In actuality, this graph shows that crime is growing at an increasing rate. If we were to graph the number of crimes as a function of the year, the graph would be concave up.

24. **(a)** From O to A, the rate is zero, so no water is flowing into the reservoir, and the volume remains constant. From A to B, the rate is increasing, so the volume is going up more and more quickly. From B to C, the rate is holding steady, but water is still going into the reservoir—it's just going in at a constant rate. So volume is increasing on the interval from B to C. Similarly, it is increasing on the intervals from C to D and from D to E. Even on the interval from E to F, water is flowing into the reservoir; it is just going in more and more slowly (the *rate* of flow is decreasing, but the total amount of water is still increasing). So we can say that the volume of water increases throughout the interval from A to F.

(b) The volume of water is constant when the rate is zero, that is from O to A.

(c) According to the graph, the rate at which the water is entering the reservoir reaches its highest value at $t = D$ and stays at that high value until $t = E$. So the volume of water is increasing most rapidly from D to E. (Be careful. The rate itself is increasing most rapidly from C to D, but the volume of water is increasing fastest when the rate is at its highest points.)

(d) When the rate is negative, water is leaving the reservoir, so its volume is decreasing. Since the rate is negative from F to I, we know that the volume of water *decreases* on that interval.

25. (a) This is a case in which the rate of decrease is constant, i.e., the change in y divided by the change in x is always the same. We see this in Table (F), where y decreases by 80 units for every decrease of 1 unit in x, and graphically in Graph (IV).

(b) Here, the change in y gets smaller and smaller relative to corresponding changes in x. In Table (G), y decreases by 216 units for a change of 1 unit in x initially, but only decreases by 6 units when x changes by 1 unit from 4 to 5. This is seen in Graph (I), where y is falling rapidly at first, but much more slowly for longer values of x.

(c) If y is the distance from the ground, we see in Table (E) that initially it is changing very slowly; by the end, however, the distance from the ground is changing rapidly. This is shown in Graph (II), where the decrease in y is larger and larger as x gets bigger.

(d) Here, y is decreasing quickly at first, then decreases only slightly for a while, then decreases rapidly again. This occurs in Table (H), where y decreases from 147 units, then 39, and finally by another 147 units. This corresponds to Graph (III).

26. (a) This describes a situation in which y is increasing rapidly at first, then very slowly at the end. In Table (E), y increases dramatically at first (from 20 to 275) but is hardly growing at all by the end. In Graph (I), y is increasing at a constant rate, while in Graph (II), it is increasing faster at the end. Graph (III) increases rapidly at first, then slowly at the end. Thus, scenario (a) matches with Table (E) and Graph (III).

(b) Here, y is growing at a constant rate. In Table (G), y increases by 75 units for every 5-unit increase in x. A constant increase in y relative to x means a straight line, that is, a line with a constant slope. This is found in Graph (I).

(c) In this scenario, y is growing at a faster and faster rate as x gets larger. In Table (F), y starts out by growing by 16 units, then 30, then 54, and so on, so Table (F) refers to this case. In Graph (II), y is increasing faster and faster as x gets larger.

27. (a) If l is the length of one salmon and its speed is u, then

$$u = 19.5\sqrt{l}.$$

Suppose the speed of the longer salmon is U and its length is $4l$. Then

$$U = 19.5\sqrt{4l} = 2 \cdot 19.5\sqrt{l} = 2u.$$

Thus, the larger one swims twice as fast as the smaller one.

(b) A typical graph is in Figure 2.51. Notice that the graphs are all of the shape of $y = \sqrt{x} = x^{1/2}$. All the graphs are increasing and concave down.

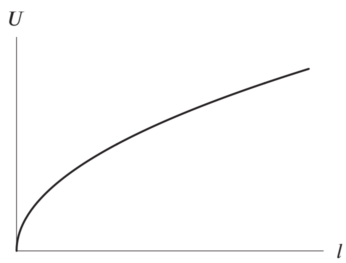

Figure 2.51

(c) The function $U = \sqrt{l}$ is an increasing function. Because $\sqrt{l}$ is an increasing function the equation predicts that larger salmon swim faster than smaller ones.

(d) The graph of $U = \sqrt{l}$ is concave down. Because the graph is concave down equal changes in l give smaller changes in U for the larger l. Thus, the difference in speed between the two smaller fish is greater than the difference in speed between the two larger fish.

28. Since f is concave down, the average rate of change is decreasing as x increases. Therefore,

$$\frac{f(3)-f(1)}{3-1} > \frac{f(5)-f(3)}{5-3}.$$

Solutions for Chapter 2 Review

Exercises

1. To evaluate when $x=-7$, we substitute -7 for x in the function, giving $f(-7) = -\frac{7}{2} - 1 = -\frac{9}{2}$.

2. To evaluate when $x=-7$, we substitute -7 for x in the function, giving $f(-7) = (-7)^2 - 3 = 49 - 3 = 46$.

3. We have

$$y = f(4) = \frac{6}{2-4^3} = \frac{6}{2-64} = \frac{6}{-62} = -\frac{3}{31}.$$

Solve for x:

$$\begin{aligned} \frac{6}{2-x^3} &= 6 \\ 6 &= 6(2-x^3) \\ 1 &= 2-x^3 \\ x^3 &= 1 \\ x &= 1. \end{aligned}$$

4. We have

$$y = f(4) = \sqrt{20+2\cdot 4^2} = \sqrt{52}.$$

Solve for x:

$$\begin{aligned} \sqrt{20+2x^2} &= 6 \\ 20+2x^2 &= 36 \\ 2x^2 &= 16 \\ x^2 &= 8 \\ x &= \pm\sqrt{8}. \end{aligned}$$

5. We have

$$y = f(4) = 4\cdot 4^{3/2} = 4\cdot 2^3 = 4\cdot 8 = 32.$$

Solve for x:

$$\begin{aligned} 4x^{3/2} &= 6 \\ x^{3/2} &= 6/4 \\ x^3 &= 36/16 = 9/4 \\ x &= \sqrt[3]{9/4}. \end{aligned}$$

6. We find:

$$y = f(4) = (4)^{-3/4} - 2 = \frac{1}{(\sqrt[4]{4})^3} - 2 = \frac{1}{(\sqrt{2})^3} - 2 = \frac{1}{\sqrt{2}\sqrt{2}\sqrt{2}} - 2 = \frac{1}{2\sqrt{2}} - 2.$$

Solve for x:

$$\begin{aligned} x^{-3/4} - 2 &= 6 \\ x^{-3/4} &= 8 \\ x = 8^{-4/3} &= \frac{1}{(\sqrt[3]{8})^4} = \frac{1}{2^4} \\ &= \frac{1}{16}. \end{aligned}$$

7. **(a)** Substituting $x = 0$ gives $f(0) = 2(0) + 1 = 1$.
(b) Setting $f(x) = 0$ and solving gives $2x + 1 = 0$, so $x = -1/2$.

8. Substituting -2 for x gives

$$f(-2) = \frac{-2}{1-(-2)^2} = \frac{-2}{1-4} = \frac{2}{3}.$$

9. Substituting 4 for t gives

$$P(4) = 170 - 4 \cdot 4 = 154.$$

Similarly, with $t = 2$,

$$P(2) = 170 - 4 \cdot 2 = 162,$$

so

$$P(4) - P(2) = 154 - 162 = -8.$$

10. **(a)** Substituting, $h(x+3) = \dfrac{1}{x+3}$.
(b) Substituting and adding, $h(x) + h(3) = \dfrac{1}{x} + \dfrac{1}{3}$.

11. **(a)** Reading from the table, we have $f(1) = 2$, $f(-1) = 0$, and $-f(1) = -2$.
(b) When $x = -1$, $f(x) = 0$.

12. **(a)** $f(0)$ is the value of the function when $x = 0$, $f(0) = 3$.
(b) $f(x) = 0$ for $x = -1$ and $x = 3$.
(c) $f(x)$ is positive for $-1 < x < 3$.

13. The expression x^2-9, found inside the square root sign, must always be non-negative. This happens when $x \geq 3$ or $x \leq -3$, so our domain is $x \geq 3$ or $x \leq -3$.
For the range, the smallest value $\sqrt{x^2-9}$ can have is zero. There is no largest value, so the range is $q(x) \geq 0$.

14. To evaluate $m(r)$, we must have $r^2 - 1 > 0$. This happens when $r > 1$ or $r < -1$. So the domain is all real numbers r, such that $r > 1$ or $r < -1$. Since the square root of a number cannot be negative, the range is all positive real numbers, $m(r) > 0$.

15. Since for any value of x that you might choose you can find a corresponding value of $m(x)$, we can say that the domain of $m(x) = 9 - x$ is all real numbers.
For any value of $m(x)$ there is a corresponding value of x. So the range is also all real numbers.

16. Since you can choose any value of x and find an associated value for $n(x)$, we know that the domain of this function is all real numbers.
However, there are some restrictions on the range. Since x^4 is always positive for any value of x, $9 - x^4$ will have a largest value of 9 when $x = 0$. So the range is $n(x) \leq 9$.

17. Since $m(t)$ is a linear function, the domain of $m(t)$ is all real numbers. For any value of $m(t)$ there is a corresponding value of t. So the range is also all real numbers.

18. Since division by 0 is undefined, $s(q)$ is not defined when $5-4q=0$. So the domain of $s(q)$ is all real numbers except $q=5/4$. To find the range we find the inverse function. We solve the equation $y=s(q)=\frac{2q+3}{5-4q}$ for q.

$$\begin{aligned} y &= \frac{2q+3}{5-4q} \\ y(5-4q) &= 2q+3 \\ 5y-4qy &= 2q+3 \\ 5y-3 &= q(2+4y) \\ q &= \frac{5y-3}{2+4y} \\ s^{-1}(y) &= \frac{5y-3}{2+4y}. \end{aligned}$$

The domain of the inverse function is all real numbers except $-1/2$, so the range of $s(q)$ is all real numbers except $-1/2$.

19. (a) If $x=a$ is not in the domain of f there is no point on the graph with x-coordinate a. For example, there are no points on the graph of the function in Figure 2.52 with x-coordinates greater than 2. Therefore, $x=a$ is not in the domain of f for any $a>2$.

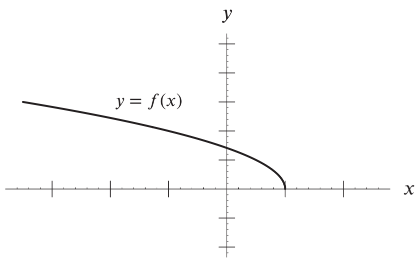

Figure 2.52

(b) If $x=a$ is not in the domain of f the formula is undefined for $x=a$. For example, if $f(x)=1/(x-3)$, $f(3)$ is undefined, so 3 is not in the domain of f.

20. (a) $-3g(x)=-3(x^2+x)$.
(b) $g(1)-x=(1^2+1)-x=2-x$.
(c) $g(x)+\pi=(x^2+x)+\pi=x^2+x+\pi$.
(d) $\sqrt{g(x)}=\sqrt{x^2+x}$.
(e) $g(1)/(x+1)=(1^2+1)/(x+1)=2/(x+1)$.
(f) $(g(x))^2=(x^2+x)^2$.

21. (a) $2f(x)=2(1-x)$.
(b) $f(x)+1=(1-x)+1=2-x$.
(c) $f(1-x)=1-(1-x)=x$.
(d) $(f(x))^2=(1-x)^2$.
(e) $f(1)/x=(1-1)/x=0$.
(f) $\sqrt{f(x)}=\sqrt{1-x}$.

22. $f(g(x))=f(x^3+1)=3(x^3+1)-7=3x^3-4$.

23. $g(f(x))=g(3x-7)=(3x-7)^3+1$.

24. $a(g(w))$ is the acceleration in meters/sec^2 when the wind speed is w meters/second.

25. $P(f(t))$ is the period, in seconds, of the pendulum at time t minutes.

26. $f(g(0))=f(2\cdot 0+3)=f(3)=3^2+1=10$.

27. $f(g(1))=f(2\cdot 1+3)=f(5)=5^2+1=26$.

28. $g(f(0))=g(0^2+1)=g(1)=2\cdot 1+3=5$.

29. $g(f(1))=g(1^2+1)=g(2)=2\cdot 2+3=7$.

30. $f(g(x)) = f(2x+3) = (2x+3)^2 + 1 = 4x^2 + 12x + 10.$

31. $g(f(x)) = g(x^2+1) + 3 = 2(x^2+1) + 3 = 2x^2 + 5.$

32. $f(f(x)) = f(x^2+1) = (x^2+1)^2 + 1 = x^4 + 2x^2 + 2.$

33. $g(g(x)) = g(2x+3) = 2(2x+3) + 3 = 4x + 9.$

34. The inverse function, $f^{-1}(V)$, gives the time at which the speed is V. Units of $f^{-1}(V)$ are seconds.

35. The inverse function, $f^{-1}(I)$, gives the interest rate that gives $\$I$ in interest. Units of $f^{-1}(I)$ is percent per year.

36. We can substitute any positive number for x, which makes the domain $x > 0$. We note that as x approaches infinity $h(x)$, approaches zero, no matter the magnitude of a. If a is positive, then as x approaches zero, $h(x)$ approaches infinity. If a is negative, then as x approaches zero, $h(x)$ approaches negative infinity. Thus, the range is all positive numbers if $a > 0$ and all negative numbers if $a < 0$. Finally if $a = 0$, the range is just 0.

37. The function is defined for all values of x. Since $|x - b| \geq 0$, the range is all numbers greater than or equal to 6.

38. Since $y = \sqrt{t} + 1$, solving for t gives

$$\begin{aligned}\sqrt{t} + 1 &= y\\ \sqrt{t} &= y - 1\\ t &= (y-1)^2\\ g^{-1}(y) &= (y-1)^2.\end{aligned}$$

39. Since $P = 14q - 2$, solving for q gives

$$\begin{aligned}14q - 2 &= P\\ q &= \frac{P+2}{14}\\ f^{-1}(P) &= \frac{P+2}{14}.\end{aligned}$$

40. The composition, $P = g(f(t))$ gives the daily electricity consumption in megawatts at time t.

41. If $P = f(t)$, then $t = f^{-1}(P)$, so $f^{-1}(P)$ gives the time in years at which the population is P million.

42. If $E = g(P)$, then $P = g^{-1}(E)$ so $g^{-1}(E)$ gives the population leading to a daily electricity consumption of E megawatts.

43. The rate of change between $t = 0.2$ and $t = 0.4$ is

$$\frac{\Delta p(t)}{\Delta t} = \frac{-2.32 - (-3.19)}{0.4 - 0.2} = 4.35.$$

Similarly, we have

$$\begin{aligned}\frac{\Delta p(t)}{\Delta t} &= \frac{-1.50 - (-2.32)}{0.6 - 0.4} = 4.10\\ \frac{\Delta p(t)}{\Delta t} &= \frac{-0.74 - (-1.50)}{0.8 - 0.6} = 3.80.\end{aligned}$$

The rate of change is decreasing, so we expect the graph to be concave down.

44. We have

$$p(8) = \frac{12}{\sqrt{8}} = \frac{6}{\sqrt{2}}.$$

If $\sqrt{2} = 12/\sqrt{x}$, then $\sqrt{2x} = 12$, so $x = 72$. Thus, $p^{-1}(\sqrt{2}) = 72$.

45. We find $f(16) = 12 - \sqrt{16} = 8$. If $3 = 12 - \sqrt{x}$, then $\sqrt{x} = 9$, so $x = 81$. Thus, $f^{-1}(3) = 81$

46. The graph of $f(x) = \begin{cases} x^2 & \text{for} \quad x \leq 1 \\ 2 - x & \text{for} \quad x > 1 \end{cases}$ is shown in Figure 2.53.

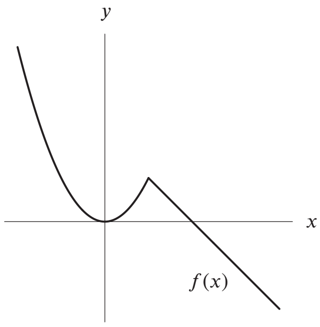

Figure 2.53

47. The graph of $g(x) = \begin{cases} x+5 & \text{for} \quad x < 0 \\ x^2+1 & \text{for} \quad 0 \le x \le 2 \\ 3 & \text{for} \quad x > 2 \end{cases}$ is shown in Figure 2.54.

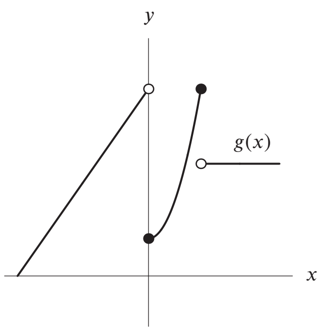

Figure 2.54

Problems

48. Substituting $r = 3$ and $h = 2$ gives

$$V = \frac{1}{3}\pi 3^2 \cdot 2 = 6\pi \text{ cubic inches.}$$

49. **(a)** Substituting, $q(5) = 3 - (5)^2 = -22$.
(b) Substituting, $q(a) = 3 - a^2$.
(c) Substituting, $q(a-5) = 3 - (a-5)^2 = 3 - (a^2 - 10a + 25) = -a^2 + 10a - 22$.
(d) Using the answer to part (b), $q(a) - 5 = 3 - a^2 - 5 = -a^2 - 2$.
(e) Using the answer to part (b) and (a), $q(a) - q(5) = (3 - a^2) - (-22) = -a^2 + 25$.

50. Substituting -1 gives $p(-1) = (-1)^2+(-1)+1 = 1$. Substituting 1 and taking the negative gives, $-p(1) = -((1)^2+(1)+1) = -3$. Thus, $p(-1) \neq -p(1)$. They are not equal.

51. The graph of g is the graph of f shifted down by 1 unit and to the left by 2 units. Thus,

$$g(x) = f(x+2) - 1.$$

52. **(a)** The table shows $f(6) = 3.7$, so $t = 6$. In a typical June, Chicago has 3.7 inches of rain.
(b) First evaluate $f(2) = 1.8$. Solving $f(t) = 1.8$ gives $t = 1$ or $t = 2$. Chicago has 1.8 inches of rain in January and in February.

53. **(a)** Substituting $x = 0$ gives $f(0) = \sqrt{0^2 + 16} - 5 = \sqrt{16} - 5 = 4 - 5 = -1$.

(b) We want to find x such that $f(x) = \sqrt{x^2 + 16} - 5 = 0$. Thus, we have

$$\begin{aligned} \sqrt{x^2+16} - 5 &= 0 \\ \sqrt{x^2+16} &= 5 \\ x^2 + 16 &= 25 \\ x^2 &= 9 \\ x &= \pm 3. \end{aligned}$$

Thus, $f(x) = 0$ for $x = 3$ or $x = -3$.

(c) In part (b), we saw that $f(3) = 0$. You can verify this by substituting $x = 3$ into the formula for $f(x)$:

$$f(3) = \sqrt{3^2 + 16} - 5 = \sqrt{25} - 5 = 5 - 5 = 0.$$

(d) The vertical intercept is the value of the function when $x = 0$. We found this to be -1 in part (a). Thus the vertical intercept is -1.

(e) The graph touches the x-axis when $f(x) = 0$. We saw in part (b) that this occurs at $x = 3$ and $x = -3$.

54. **(a)** In order to find $f(0)$, we need to find the value which corresponds to $x = 0$. The point $(0, 24)$ seems to lie on the graph, so $f(0) = 24$.

(b) Since $(1, 10)$ seems to lie on this graph, we can say that $f(1) = 10$.

(c) The point that corresponds to $x = b$ seems to be about $(b, -7)$, so $f(b) = -7$.

(d) When $x = c$, we see that $y = 0$, so $f(c) = 0$.

(e) When your input is d, the output is about 20, so $f(d) = 20$.

55. **(a)** Looking from left to right, the graph appears to be rising between $x = -2.9$ and $x = 0$, and again between $x = 1.4$ and $x = 3$. Therefore, we estimate that f is increasing on $-2.9 < x < 0$ and on $1.4 < x < 3$.

(b) The rate of change of f appears to be increasing between $x = -4$ and $x = -1.8$, and again between $x = 0.8$ and $x = 3$. Therefore, we estimate that f is concave up on $-4 < x < -1.8$ and on $0.8 < x < 3$.

56. **(a)** Since the vertical intercept of the graph of f is $(0, 2)$, we have $f(0) = 2$.

(b) Since the horizontal intercept of the graph of f is $(-3, 0)$, we have $f(-3) = 0$.

(c) The function f^{-1} goes from y-values to x-values, so to evaluate $f^{-1}(0)$, we want the x-value corresponding to $y = 0$. This is $x = -3$, so $f^{-1}(0) = -3$.

(d) Solving $f^{-1}(?) = 0$ means finding the y-value corresponding to $x = 0$. This is $y = 2$, so $f^{-1}(2) = 0$.

57. See Figure 2.55.

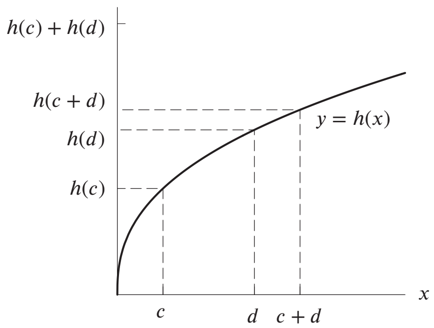

Figure 2.55

58. The input values in the table of values of g^{-1} are the output values for g. See Table 2.8.

Table 2.8

y	7	12	13	19	22
$g^{-1}(y)$	1	2	3	4	5

59. We solve the equation $V = f(r) = \frac{4}{3}\pi r^3$ for r. Divide both sides by $\frac{4}{3}\pi$ and then take the cube root to get

$$r = \sqrt[3]{\frac{3V}{4\pi}}.$$

So

$$f^{-1}(V) = \sqrt[3]{\frac{3V}{4\pi}}.$$

60. **(a)** To write s as a function of A, we solve $A = 6s^2$ for s

$$s^2 = \frac{A}{6} \qquad \text{so} \qquad s = f(A) = +\sqrt{\frac{A}{6}} \qquad \text{because the length of a side of a cube is positive.}$$

The function f gives the side of a cube in terms of its area A.

(b) Substituting $s = f(A) = \sqrt{A/6}$ in the formula $V = g(s) = s^3$ gives the volume, V, as a function of surface area, A,

$$V = g(f(A)) = s^3 = \left(\sqrt{\frac{A}{6}}\right)^3.$$

61. **(a)** Since the deck is square, $f(s) = s^2$.

(b) Since a can costs \$29.50 and covers 200 ft^2, we know

$$\text{Cost of stain for 1 ft}^2 = \frac{29.50}{200} = 0.1475 \text{ dollars.}$$

Thus

$$\text{Cost of stain for } A \text{ ft}^2 = 0.1475A \text{ dollars,}$$

so

$$C = g(A) = 0.1475A.$$

(c) Substituting for $A = f(s) = s^2$ into g gives

$$C = g(f(s)) = 0.1475s^2.$$

The function $g(f(s))$ gives the cost in dollars of staining a square deck of side s feet.

(d) (i) $f(8) = 8^2 = 64$ square feet; the area of a deck of side 8 feet.

(ii) $g(80) = 0.1475 \cdot 80 = 11.80$ dollars; the cost of staining a deck of area 80 ft^2.

(iii) $g(f(10)) = 0.1475 \cdot 10^2 = 14.75$ dollars; the cost of staining a deck of side 10 feet.

62. **(a)** This is the fare for a ride of 3.5 miles. $C(3.5) \approx \$6.25$.

(b) This is the number of miles you can travel for \$3.50. Between 1 and 2 miles the increase in cost is \$1.50. Setting up a proportion, we have:

$$\frac{\text{1 additional mile}}{\text{\$1.50 additional fare}} = \frac{x \text{ additional miles}}{\text{\$3.50} - \text{\$2.50 additional fare}}$$

and $x = 0.67$ miles. Therefore

$$C^{-1}(\$3.5) \approx 1.67.$$

63. **(a)** $P = f(s) = 4s$.

(b) $f(s+4) = 4(s+4) = 4s+16$. This the perimeter of a square whose side is four meters larger than s.

(c) $f(s) + 4 = 4s + 4$. This is the perimeter of a square whose side is s, plus four meters.

(d) Meters.

64. **(a)** Using Pythagoras' Theorem, we see that the diagonal d is given in terms of s by

$$d^2 = 2s^2$$
$$s = \sqrt{\frac{d^2}{2}} = \frac{d}{\sqrt{2}}$$
$$s = f(d) = \frac{d}{\sqrt{2}}.$$

(b) $A = g(s) = s^2$.

(c) Substituting $s = d/\sqrt{2}$ in g gives

$$A = g(s) = \left(\frac{d}{\sqrt{2}}\right)^2 = \frac{d^2}{2}.$$

(d) The function h is the composition of f and g, with f as the inside function, that is $h(d) = g(f(d))$.

65. (a) Since $f(2) = 3$, $f^{-1}(3) = 2$.

(b) Unknown

(c) Since $f^{-1}(5) = 4$, $f(4) = 5$.

66. (a) To find a point on the graph $k(x)$ with an x-coordinate of -2, we substitute -2 for x in the formula for $k(x)$. We obtain $k(-2) = 6 - (-2)^2 = 6 - 4 = 2$. Thus, we have the point $(-2, k(-2))$, or $(-2, 2)$.

(b) To find these points, we want to find all the values of x for which $k(x) = -2$. We have

$$\begin{aligned} 6 - x^2 &= -2 \\ -x^2 &= -8 \\ x^2 &= 8 \\ x &= \pm 2\sqrt{2}. \end{aligned}$$

Thus, the points $(2\sqrt{2}, -2)$ and $(-2\sqrt{2}, -2)$ both have a y-coordinate of -2.

(c) Figure 2.56 shows the desired graph. The point in part (a) is $(-2, 2)$. We have called this point A on the graph in Figure 2.56. There are two points in part (b): $(-2\sqrt{2}, -2)$ and $(2\sqrt{2}, -2)$. We have called these points B and C, respectively, on the graph in Figure 2.56.

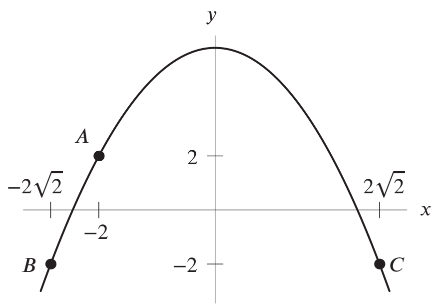

Figure 2.56

(d) For $p = 2$, $k(p) - k(p-1) = k(2) - k(1)$. Now $k(2) = 6 - 2^2 = 6 - 4 = 2$, while $k(1) = 6 - (1)^2 = 6 - 1 = 5$, thus, $k(2) - k(1) = 2 - 5 = -3$.

67. (a) To find a point on the graph of $h(x)$ whose x-coordinate is 5, we substitute 5 for x in the formula for $h(x)$. $h(5) = \sqrt{5+4} = \sqrt{9} = 3$. Thus, the point $(5, 3)$ is on the graph of $h(x)$.

(b) Here we want to find a value of x such that $h(x) = 5$. We set $h(x) = 5$ to obtain

$$\begin{aligned} \sqrt{x+4} &= 5 \\ x + 4 &= 25 \\ x &= 21. \end{aligned}$$

Thus, $h(21) = 5$, and the point $(21, 5)$ is on the graph of $h(x)$.

(c) Figure 2.57 shows the desired graph. The point in part (a) is $(5, h(5))$, or $(5, 3)$. This point is labeled A in Figure 2.57. The point in part (b) is $(21, 5)$. This point is labeled B in Figure 2.57.

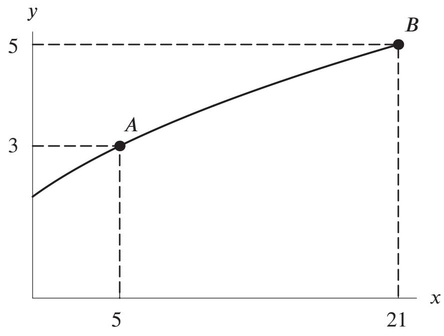

Figure 2.57

(d) If $p = 2$, then $h(p+1) - h(p) = h(2+1) - h(2) = h(3) - h(2)$. But $h(3) = \sqrt{3+4}$, while $h(2) = \sqrt{2+4}$, thus, $h(p+1) - h(p)$ for $p = 2$ equals $h(3) - h(2) = \sqrt{7} - \sqrt{6} \approx 0.1963$.

68. **(a)** $t(400) = 272$.

(b) (i) It takes 136 seconds to melt 1 gram of the compound at a temperature of 800°C.

(ii) It takes 68 seconds to melt 1 gram of the compound at a temperature of 1600°C.

(c) This means that $t(2x) = t(x)/2$, because if x is a temperature and $t(x)$ is a melting time, then $2x$ would be double this temperature and $t(x)/2$ would be half this melting time.

69. **(a)**

n	1	2	3	4	5	6	7	8	9	10	11	12
$f(n)$	1	1	2	3	5	8	13	21	34	55	89	144

(b) We note that for every value of n, we can find a unique value for $f(n)$ (by adding the two previous values of the function). This satisfies the definition of function, so $f(n)$ is a function.

(c) Using the pattern, we can figure out $f(0)$ from the fact that we must have

$$f(2) = f(1) + f(0).$$

Since $f(2) = f(1) = 1$, we have

$$1 = 1 + f(0),$$

so

$$f(0) = 0.$$

Likewise, using the fact that $f(1) = 1$ and $f(0) = 0$, we have

$$f(1) = f(0) + f(-1)$$
$$1 = 0 + f(-1)$$
$$f(-1) = 1.$$

Similarly, using $f(0) = 0$ and $f(-1) = 1$ gives

$$f(0) = f(-1) + f(-2)$$
$$0 = 1 + f(-2)$$
$$f(-2) = -1.$$

However, there is no obvious way to extend the definition of $f(n)$ to non-integers, such as $n = 0.5$. Thus we cannot easily evaluate $f(0.5)$, and we say that $f(0.5)$ is undefined.

70. **(a)** Note that each value of $a(t)$ is 0.5 greater than the value of $g(t)$ for the same t. For example, $a(0) = g(0) + 0.5$, and in general

$$a(t) = g(t) + 0.5.$$

(b) Observe that the values for $g(t)$ have been shifted to the left. For example, $b(-1) = g(0)$ and $b(0) = g(1)$. Thus,

$$b(t) = g(t+1).$$

(c) In this case, it is easier first to compare $c(t)$ and $b(t)$. For each t, the value of $c(t)$ is 0.3 less than the value of $b(t)$, so a possible formula is $c(t) = b(t) - 0.3$. Since $b(t) = g(t+1)$, we can say that

$$c(t) = g(t+1) - 0.3.$$

(d) The function $d(t)$ has the same values as $g(t)$ except they are shifted to the right by 0.5. For example, $d(0) = g(-0.5) = g(0-0.5)$ and $d(1) = g(0.5) = g(1-0.5)$. In each case

$$d(t) = g(t-0.5).$$

(e) Compare values of $e(t)$ and $d(t)$. For any value of t, $e(t)$ is 1.2 more than $d(t)$. Thus, $e(t) = d(t) + 1.2$. But, $d(t) = g(t-0.5)$, so

$$e(t) = g(t-0.5) + 1.2.$$

71. **(a)** This tells us that a person who loses 30 minutes of sleep takes 5 minutes longer to complete the task than a person who loses no sleep.

(b) This tells us that a person who loses t_1 minutes of sleep takes twice as long to complete the task as a person who loses no sleep.

(c) This tells us that a person who loses $2t_1$ minutes of sleep takes 50% longer to complete the task as a person who loses t_1 minutes of sleep.

(d) This tells us that a person who loses $t_2 + 60$ minutes of sleep takes 10 minutes longer to complete the task than a person who loses only $t_2 + 30$ minutes of sleep.

72. One way to do this is to combine two operations, one of which forces x to be non-negative, the other of which forces x not to equal 3. One possibility is

$$y = \frac{1}{x-3} + \sqrt{x}.$$

The fraction's denominator must not equal 0, so x must not equal 3. Further, the input of the square root function must not be negative, so x must be greater than or equal to zero. Other possibilities include

$$y = \frac{\sqrt{x}}{x-3}.$$

73.
- A function such as $y = \sqrt{x-4}$ is undefined for $x < 4$, because the input of the square root operation is negative for these x-values.
- A function such as $y = 1/(x-8)$ is undefined for $x = 8$.
- Combining two functions such as these, for example by adding or multiplying them, yields a function with the required domain. Thus, possible formulas include

$$y = \frac{1}{x-8} + \sqrt{x-4} \qquad \text{or} \qquad y = \frac{\sqrt{x-4}}{x-8}.$$

74. **(a)** Each signature printed costs \$0.14, and in a book of p pages, there are at least $p/16$ signatures. In a book of 128 pages, there are

$$\frac{128}{16} = 8 \text{ signatures},$$
$$\text{Cost for 128 pages } = 0.14(8) = \$1.12.$$

A book of 129 pages requires 9 signatures, although the ninth signature is used to print only 1 page. Therefore,

$$\text{Cost for 129 pages } = \$0.14(9) = \$1.26.$$

To find the cost of p pages, we first find the number of signatures. If p is divisible by 16, then the number of signatures is $p/16$ and the cost is

$$C(p) = 0.14\left(\frac{p}{16}\right).$$

If p is not divisible by 16, the number of signatures is $p/16$ rounded up to the next highest integer and the cost is 0.14 times that number. In this case, it is hard to write a formula for $C(p)$ without a symbol for "rounding up."

(b) The number of pages, p, is greater than zero. Although it is possible to have a page which is only half filled, we do not say that a book has $124\frac{1}{2}$ pages, so p must be an integer. Therefore, the domain of $C(p)$ is $p > 0$, p an integer. Because the cost of a book increases by multiples of \$0.14 (the cost of one signature), the range of $C(p)$ is $C > 0$, C an integer multiple of \$0.14.

(c) For 1 to 16 pages, the cost is \$0.14, because only 1 signature is required. For 17 to 32 pages, the cost is \$0.28, because 2 signatures are required. These data are continued in Table 2.9 for $0 \le p \le 128$, and they are plotted in Figure 2.58. A closed circle represents a point included on the graph, and an open circle indicates a point excluded from the graph. The unbroken lines in Figure 2.58 suggest, erroneously, that *fractions* of pages can be printed. It would be more accurate to draw each step as 16 separate dots instead of as an unbroken line.

Table 2.9 *The cost C for printing a book of p pages*

p, pages	C, dollars
1-16	0.14
17-32	0.28
33-48	0.42
49-64	0.56
65-80	0.70
81-96	0.84
97-112	0.98
113-128	1.12

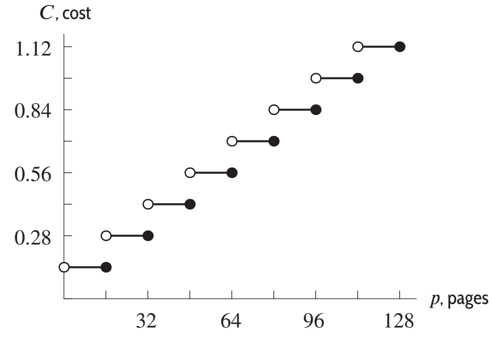

Figure 2.58: Graph of the cost C for printing a book of p pages

75. **(a)** Increasing until year 60 and then decreasing.

(b) The average rate of change of the population is given in Table 2.10.

Table 2.10 *The population of Ireland from 1780 to 1910*

Year (years)	Population (millions)	$\Delta P/\Delta t$ (millions/year)
0 to 20	4.0 to 5.2	0.060
20 to 40	5.2 to 6.7	0.075
40 to 60	6.7 to 8.3	0.080
60 to 70	8.3 to 6.9	−0.140
70 to 90	6.9 to 5.4	−0.075
90 to 110	5.4 to 4.7	−0.035
110 to 130	4.7 to 4.4	−0.015

(c) The average rate of change is increasing until between years 40 and 60. At year 60, the sign abruptly changes, but after 60, the rate of change is still increasing. Thus, the graph is concave up, although something strange is happening near year 60.

(d) The rate of change of the population was greatest between 40 and 60, that is 1820–1840. The rate of change of the population was least (most negative) between 60 and 70, that is 1840–1850. At this time the population was shrinking fastest.

Since the greatest rate of increase was directly followed by the greatest rate of decrease, something catastrophic must have happened to cause the population not only to stop growing, but to start shrinking.

(e) Figures 2.59 and 2.60 show the population of Ireland from 1780 to 1910 as a function of the time with two different curves dashed in—either of which could be correct. From the graphs we can see that the curve is increasing until about year 60, and then decreases. Also, it is concave up most of the time except, possibly, for a short time interval near year 60.

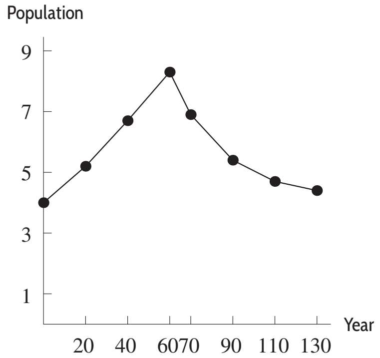

Figure 2.59: The population of Ireland from 1780 to 1910

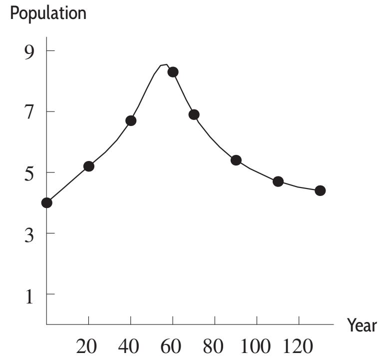

Figure 2.60: The population of Ireland from 1780 to 1910

(f) Something catastrophic happened in Ireland about year 60—that is, 1840. This is when the Irish potato famine took place.

STRENGTHEN YOUR UNDERSTANDING

1. False. $f(2) = 3 \cdot 2^2 - 4 = 8$.

2. True. Functions are evaluated by substituting a known value or variable, here b, for the independent variable, here x.

3. False. $f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$.

4. True. If $q = 1/\sqrt{z^2+5} = 1/3$, then

$$\begin{aligned}\frac{1}{z^2+5} &= \frac{1}{9}\\ z^2+5 &= 9\\ z^2 &= 4\\ z &= \pm 2.\end{aligned}$$

5. False. $W = (8+4)/(8-4) = 3$.

6. True. $f(0) = 0^2 + 64 = 64$.

7. False. For example, if $f(x) = x - 3$, then $f(x) = 0$ for $x = 3$ but not for $x = 0$.

8. False. For example, $f(1) = 10$ but $f(-1) = 6$.

9. True. A fraction can only be zero if the numerator is zero.

10. False. $h(3) + h(4) = (-6 \cdot 3 + 9) + (-6 \cdot 4 + 9) = -9 + (-15) = -24$ but $h(7) = -6 \cdot 7 + 9 = -33$.

11. True. This is the definition of the domain.

12. True. This is the common practice.

13. False. The domain consists of all real numbers x, $x \neq 3$

14. False. The domain consists of all real numbers $x \leq 2$.

15. False. The range does not include zero, since $1/x$ does not equal zero for any x.

16. False. If $x < 0$, then $y > 4$.

17. True. Since f is an increasing function, the domain endpoints determine the range endpoints. We have $f(15) = 12$ and $f(20) = 14$.

18. True. The $x^2 + 1$ inside the square root is positive for all x so $f(x)$ is defined for all x.

19. True. It has a slope of -1 to the left of the origin, goes through the origin, and continues as an increasing function with slope 1 to the right of the origin.

20. True. It is defined for all x.

21. True. $|x| = |-x|$ for all x.

22. False. For example, if $x = 1$ then $f(1) = 1$ and $g(1) = -1$.

23. False. For example, if $x = -1$ then $y = -1$.

24. False. Since $0 \le 3 < 4$, the middle formula must be used. So $f(3) = 3^2 = 9$.

25. True. If $x < 0$, then $f(x) = x < 0$, so $f(x) \neq 4$. If $x > 4$, then $f(x) = -x < 0$, so $f(x) \neq 4$. If $0 \le x \le 4$, then $f(x) = x^2 = 4$ only for $x = 2$. The only solution for the equation $f(x) = 4$ is $x = 2$.

26. True. The graph of $g(x)$ is a copy of the graph of f shifted vertically up by three units.

27. False. The horizontal shift is two units to the right.

28. False. In the figure in the problem, it appears that $g(x) = f(x-2) + 1$ because the graph is two units to the right and one unit up from the graph of f.

29. True. This looks like the absolute value function shifted right 1 unit and down 2 units.

30. False. If f is invertible, we know $f^{-1}(5) = 3$, but nothing else.

31. True. This is the definition of the inverse function.

32. False. Check to see if $f(0) = 8$, which it does not.

33. True. To find $f^{-1}(R)$, we solve $R = \frac{2}{3}S + 8$ for S by subtracting 8 from both sides and then multiplying both sides by $(3/2)$.

34. False. For example, if $f(x) = x + 1$ then $f^{-1}(x) = x - 1$ but $(f(x))^{-1} = \dfrac{1}{x+1}$.

35. True. Since $t^{-1} = 1/t$ this is a direct substitution for the independent variable x.

36. False. The output units of a function are the same as the input units of its inverse.

37. False. Since

$$f(g(x)) = 2\left(\frac{1}{2}x - 1\right) + 1 = x - 1 \neq x,$$

the functions do not undo each other.

38. True. The quantity of rice required is $f(x)$ tons. The cost of this quantity is $g(f(x))$ dollars. Thus, the cost to feed x million people for a year is $g(f(x))$ dollars.

39. True.

40. False. The composite function $g(f(t))$ gives the volume of the ball in meter^3 after t seconds. Thus, the units of $g(f(t))$ are meter^3.

41. True. Since the function is concave up, the average rate of change increases as we move right.

42. True. The rates of change are increasing:

$$\frac{f(0) - f(-2)}{0 - (-2)} = \frac{6-5}{2} = \frac{1}{2}.$$

$$\frac{f(2) - f(0)}{2 - 0} = \frac{8-6}{2} = 1.$$

$$\frac{f(4) - f(2)}{4 - 2} = \frac{12-8}{2} = 2.$$

43. True. The rates of change are decreasing:

$$\frac{g(1) - g(-1)}{1 - (-1)} = \frac{8-9}{2} = -\frac{1}{2}.$$

$$\frac{g(3) - g(1)}{3 - 1} = \frac{6-8}{2} = -1.$$

$$\frac{g(5) - g(3)}{5 - 3} = \frac{3-6}{2} = -\frac{3}{2}.$$

44. False. A straight line is neither concave up nor concave down.

45. True. For $x > 0$, the function $f(x) = -x^2$ is both decreasing and concave down.

46. False. For $x < 0$, the function $f(x) = x^2$ is both concave up and decreasing.